THE MATHEMATICAL
ASSOCIATION OF VICTORIA

INVESTIGATIONS

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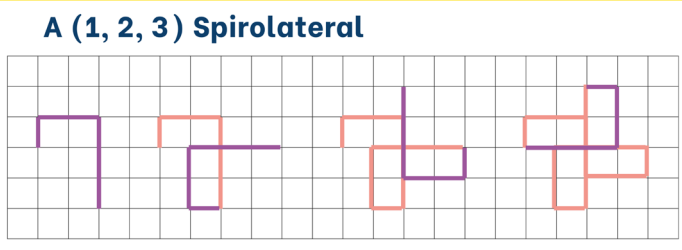
SPIROLATERALS

Starting with 3 numbers and following a path around a grid, something interesting emerges. It's known as a Spirolateral. This rich investigation offers a beautiful example of mathematical connections, and will repeatedly open new avenues to explore.



HOW IT WORKS

This is a (1, 2, 3) Spirolateral. It's made by moving:

1. One space up.
2. Turning 90° clockwise. Moving 2 spaces to the right.
3. Turning 90° clockwise. Moving 3 spaces down.
4. Repeating steps 1-3 (turning 90° and moving 1, 2 or 3 spaces repeatedly).

Show students how a few 3-number Spirolaterals are made. Invite observations. Note: the 3 numbers do not need to be consecutive, or in increasing order. There can be repeats.

Spirolaterals like (1, 2, 3) meet at a point in the middle. While others, like (3, 1, 5), form a square. The main challenge:

Pick any three numbers to build a spirolateral. Can you predict if the lines will meet at a point in the middle? Or form a square? Can you explain why?

Allow students time to play, explore, ask, guess, look for patterns and explain. Potential strategies:

- Use the same 3 numbers in different positions.
- Keep 2 numbers the same, and change the third.
- Create a list of Spirolaterals that meet at a point vs form a square.

Wrap up by discussing solutions, as well as strategies they used to create different kinds of Spirolaterals and to organise data.

A key insight students may notice is that when 2 numbers add to the third number, e.g. (1, 3, 4), the lines meet at a point in the middle (i.e. there is no square). But, when 2 numbers add to less than the third number, e.g. (1, 3, 5), or more than the third number, e.g. (3, 5, 7), a square is formed. A good question to ask here is: 'Why does this happen?!'

GO DEEPER

Some prompts to help students take their thinking further, as they explore this problem:

- How might you keep track of what you've done?
- Consider these Spirolaterals: e.g. (2, 3, 4) and (3, 2, 4). What's the same/different? What about (1, 1, 2) and (1, 2, 2)?
- Describe how to create a Spirolateral with a point/square.
- Will the Spirolaterals always go back to the start? Why?
- What are you (un)sure of? What might be your next step?

WHY USE IT?

This task provides learners valuable practice in:

- Understanding symmetry and transformations
- Looking for patterns and using algorithms
- Spatial reasoning
- Organising and making sense of data
- Making and breaking conjectures.

VC:M 2.0 LINKS

Level 1: VC2M1SP02, Level 2: VC2M2SP02, Level 3: VC2M3N09, Level 4: VC2M4N10, VC2M5N10, VC2M5SP03. Level 5: VC2M6A03, Level 6: VC2M6SP03

Scan this QR code to get a full lesson plan that includes solutions and variations of the task:

What kinds of investigations have you used in your classroom as a launch for mathematical exploration? Our readers would love to hear your experiences. You can share your ideas with us at primenumber@mav.vic.edu.au.