THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR

4/20

DITCH, GROW, AMPLIFY

INSIDE

Martin Holt - Primary mathematics education consultant. Illustration by Dominic Creswell-White.

WHAT I LEARNT LAST TERM

I won't go on about how extraordinary Term 3 was. Instead, I want to reflect on my experience as an educational consultant working with the Mathematical Association of Victoria. I hope this article provide insights on the successes and challenges in driving innovation in maths teaching and learning in schools. This article may be of interest if you are currently thinking about building your own teaching capacity or looking at ways to drive whole-school improvement that is responsive to remote or more traditional learning.

To assess what worked and what was less successful, I will use one of my favourite exit tickets, called Ditch, Grow, Amplify. Ditch is self-explanatory. Grow is in reference to a new idea that I want to implement. Amplify means something that I already do that I want to promote

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THE COMMON DENOMINATOR
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FROM THE PRESIDENT

Michael O'Connor

ENGAGING MATHEMATICS – MY STORY

MAV is going virtual!

At least our December Conference is. And I think it is a wonderful decision, turning a challenge into an opportunity.

In the last couple of years, we have begun exploring broadcasting a few of the keynote sessions and talking at staff and Board level about what else we might offer. Faced, now, with the reality that is 2020 the transition from talk to action has been swift and decisive.

What I am most excited about with this decision is that it offers tangible benefits to the mathematics teaching community of Victoria. For over twenty years MAV has been the largest mathematics teachers conference in Australia, with consistent attendances at or above 1400 participants. We know, though, that this is just a small fraction of the teachers in the state. By providing an online event that records all the presentations we have the potential to reach a hundred percent of those whom we serve. So please, if you have not already, read through the synopsis and join us on December 3 and 4.

In the synopsis welcome to the conference, Ann Downton asks two questions. 'What is your story?' and 'How do you engage in mathematics?' In pausing to think about those questions I am reminded of my two things my dad told me, or rather one thing he told me and something he showed me, when I was growing up.

My family were sheep farmers and several times a year we would bring the mobs into the yards for shearing, drenching and the like. Every time we brought them in, we would have to count them as the raced down the race, a narrow open tunnel between pens. As I am sure you can imagine, sheep don't tend to stand still and it is very hard to count them one by one. Dad once asked my brother and I, 'How do you count them?' we looked somewhat blankly back at him so he continued, 'You need to do it in twos and threes at a time.' What I discovered many years later was that this can only be done if you subitise the pairs and trios. One of the very first elements of learning mathematics at school is precisely this same skill.

The other occasion was on a day when we were building or repairing something around the house. We had the tools and timber out. Dad picked up a length of wood, a couple of nails and a loop of string. He then proceeded to use them to draw an ellipse on the timber. There was an elegant simplicity in the materials he used and the construct he conjured from them. I was fascinated!

No matter what our age, five or fifty, being able to experience the joy, to engage in conjuring visions from nothingness or realising a new and ingenious way of completing a task, is something that as teachers we should yearn for and strive to pass on to those that follow.

MAV20 GOES VIRTUAL

Peter Saffin, CEO – Mathematical Association of Victoria

The last few months have been a period of change and challenge for all of us and in particular teachers throughout Australia.

Like many associations we considered cancelling our annual conference due to the uncertainty surrounding COVID-19. However, with the support of MAV Board, MAV events team and the conference committee I am pleased to announce the 57th Annual Virtual Conference (MAV20) will be held on Thursday 3 and Friday 4 December 2020.

You can access all 100 sessions for 12 months post-conference including presentations, publications and resources. When you usually get to see only a few sessions per day, you can now go back and watch them all!

Another benefit of this virtual event is that we expect more regional educators will be able to attend, with reduced costs and no travel time. We are pleased to offer an event that levels the playing field for educators across Victoria.

MAV is investing its time and resources to deliver a highly practical, engaging and professional event using an external virtual event provider to ensure a no-fuss, easy to access and socially engaging experience. We look forward to seeing you there with your community of mathematics educators from across Victoria, Australia and the world. We need you to make MAV20 a success, that part is not new!

MAV2020 THEME

We all experience mathematics differently; some of us are confident, others suffer from maths anxiety, and many of us sit between these two extremes. What is your story? How do you engage in mathematics?

How can we as a community: ignite students' passion for, and value of, mathematics; engage and support colleagues' professional growth; and, engage and inform the community as a whole? Let's create a 2020 vision to share the positive skills and attributes that mathematics can provide for future generations.

5 reasons to attend

- 10 awesome live keynote choices including two panel sessions
- Plus 90 sessions over 2 days with interactive Q&A
- You've already saved the date, it takes place over the planned two days in December
- Enjoy networking lunch, happy hour and competitions
- Support our sponsors and exhibitors who are ready to take part in our virtual exhibition.

For a copy of the program and to book visit: https://conference.mav.vic.edu.au

DITCH, GROW, AMPLIFY

Martin Holt - Primary mathematics education consultant

CONT. FROM PAGE 1.

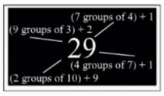
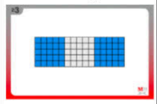
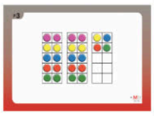
Sequence of Learning	**Learning focus: Equal groups, division with remainder.**	Equal groups concrete materials	Equal groups concrete materials	Equal groups - using movement	Division with remainder - Add to sequence - sharing as repeated subtraction - Represent division using shape as fractional part - Need to inc number lines, 100s charts.			
Hooks / Warm-ups	Sticky Labels 1. Roll a 10-sided die twice. The first number shows the number of groups, the second shows the number in each group. A roll of 7 then 6 would be 7 groups of 6. Write this on a post-it and stick it on the forehead of a fellow student. Find out what your number is by asking yes/no questions.	Sticky Labels 2. Repeat the activity but this time have students draw the groups and the number of each group. E.g. a roll of 7 and 6 would show 7 hoops with 6 dots inside each hoop.	Sorting Sticky Labels Layout all o the sticky labels from the previous two lessons. Work as a class to connect the symbolic and pictorial representations.	Back-to-back Drawing. Write a number on the board. Have student A describe to Student B how to draw that number in equal rows and columns (an array). Use a visual such as apples to represent.	Number Talk Can you work out how many? How did you solve it? Is there a remainder? Record student thinking on the board. 	Bob's Buttons Have students move around the room to music. Stop the music and ask students to make groups of 4. Record the different groups and remainders on the board. Repeat using a different number. 	Paper Plates - Hold up two plates. Plate 1 shows the groups, plate 2 shows the number in each group. Draw what this could look like visually i.e an array, cubes and hoops. 	Number Talk Can you work out how many? How did you solve it? Who worked it out in a similar way? Who used a different strategy? 
Lessons	Pre-assessment - Animal Legs: Mathematics Assessment for Learning p. 25. 	Pic Story Book The Doorbell Rang - investigate different ways to share 24 cookies. 	The chef had 24 pizzas in trays in the oven. There is the same number of pizzas on each tray. What might this look like?	18 people said they wanted to do folk dancing. The teacher said they must dance in groups. How many different types of groups could they make? Explore the problem using physical movement and record thinking using a number line.	Share 59 icy-pole sticks among 4 people. Explore the different strategies used.	Bob's Buttons - Maths 300 Martin Holt Lesson Plan	Bob's Buttons - Maths Task Centre - follow-up task. 	Open-ended task. On Wednesday I gave away all of my footy cards to five of my friends. After I shared them equally, I had 3 cards left over. How many cards might I have started with?

Figure 1. The big mathematical idea for this sequence: When we make equal groups, there is sometimes a remainder.

even more. These prompts are a 'call to action' which is a popular and effective strategy used in advertising to mobilise people. More than this, they connote an expectation that regardless of perceived successes and failures, the respondent will continue to look for ways to learn and improve.

THINGS I PLAN TO DITCH

The bunker mentality

I went down the 'wait-it-out' route for lockdown 1.0. I spent my time working on pet projects that were more conducive to an isolated life. Like many other Victorians, lockdown 2.0 was a body-blow. I realised that I had to take a more proactive approach to professional learning. I am grateful for the support of Jen Bowden and Ellen Corovic at MAV for pioneering this mentality. Ditto to the maths leadership teams at Toorak and Heidelberg Primary School who have not let a global pandemic stop them from treading the path of continuous improvement.

Bad practice is bad practice

There were been times during Term 3 when my online coaching was caught up in delivering ideas and not letting teaching teams spend time developing ideas for themselves. Most of the time, this happened because I fell into the trap of wanting to give teachers as much 'stuff' in an as little time as possible. This approach invariably creates a sieve effect because teachers don't feel invested in the process. The same thing happens with our students. Regardless of the different ways, we interact these days, learning by doing continues to be just about the most effective way to learn. Since those early mistakes, I have made sure that every planning session provides time for teachers to explore some of the tasks for themselves.

THINGS I'LL GROW

Promoting learning sequences

I love how learning sequences can take a concept and make it stick by having it show up again and again through a series of lessons. Like an old friend, it might look a bit different, but deep down it hasn't changed. Learning sequences require a bit of heavy lifting at the start. A lot of work goes into identifying the big mathematical ideas that drive the learning. Tasks are selected and sequenced according to their capacity to move students along a learning trajectory where they understand a big mathematical idea in increasingly sophisticated ways. Instead of detailing these here, I have provided two excellent articles at the end of the article. Figure 1 shows draft excerpt from a division learning sequence that I co-developed with the Year 2 team at Heidelberg Primary School.

During the session, we discussed different ways to adjust tasks for remote learning by:

- using the chat function as one way to share and monitor student thinking
- using Seesaw or similar as a platform to showcase different strategies used, address common misconceptions.
- having students source their own counters to explore and upload their thinking such as sorting pasta shells into equal groups during the Maths 300 investigation Bob's Buttons.

Tweaking the Launch, Explore, Summarise instructional model for remote learning

Faced with the logistical challenges of sending students away and trying to get them back, teachers that I have worked with have had success breaking the whole, part, whole instructional model over two days. In practice, teachers spend the first 10-15 minutes conducting the lesson summary from the previous day before launching the next lesson right after. The benefit of this is that it gives students time to grapple with the problem independently knowing that they will have an opportunity to come together the following day to consolidate their understanding. This approach works especially well when teaching learning sequences as the summary of one lesson makes an excellent segway for the next lesson. Students that might have just seen one of their peers demonstrate an efficient strategy have an opportunity to apply it almost straight away.

THINGS I'LL AMPLIFY

Number talks

Number talks have been a winner during remote learning. Consider some of the possibilities for mathematical reasoning, fluency and understanding when students are given the opportunity to spend time thinking about different ways to solve these mathematical representations. Thumbs up when you've found one way to solve it. Use fingers if you can find more than one way. Who solved it a similar way? Who used a different strategy?

More coaching in teams

The most effective way to build teacher capacity is to work with individual teaching teams. This is enhanced when teams have a specific challenge or goal that they want to focus on. Follow-up is also important. Two weeks is the perfect amount of time to have a go at teaching a learning sequence for the first time, or perhaps further embedding enabling and extending prompts or putting greater emphasis on the summary phase of the lesson. It is clear that teaching teams are driving momentum for whole-school improvement under this model.

I am excited by the prospect of supporting schools in developing a shared vision of what they want maths teaching and learning to look like in their classrooms. As part of this process, we will consider if there are things that they are doing that should be ditched. What ideas do we want to grow and what should be amplified? Once this vision is articulated, we'll work on measurable implementation steps for teachers.

REFERENCES

Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., Russo, J., & Sullivan, P. (2018). Curriculum documentation and the development of effective sequences of learning experiences. In *International Commission on Mathematical Instruction (ICMI) Study Conference 2018* (pp. 499-506). International Commission on Mathematical Instruction (ICMI).

Sullivan, P., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Russo, J. (2020). Ways that relentless consistency and task variation contribute to teacher and student mathematics learning. In *For the Learning of Mathematics: Proceedings of a symposium on learning in honour of Laurinda Brown: Monograph 1* (pp. 32-37). FLM Publishing Association.

MAV can build capacity through in-school consulting and professional learning. For more info, contact Jen Bowden, jbowden@mav.vic.edu.au

PREPARING FOR VCE EXAMS

Helen Haralambous, Mary Papp, Allason McNamara, Sue Garner and Fiona Latrobe

The first thing to remember is that 2020 is a unique time in which both students and teachers have had a mixed year due to COVID-19. Some teaching has been done using learning-at-home and some face-to-face. Your teachers will have done their absolute best to prepare you and the exams will provide an equal platform as all Victorian students have faced the same constrictions. In all three mathematics studies, changes have been made and a unique 2020 Adjusted Study Design has been produced to cater to this. For the full details refer to https://www.vcaa.vic.edu.au/Documents/vce/adjustedSD2020/2020MathematicsSD.pdf

The best preparation for exams is completing and marking as many past exams as you can. These are freely available on the VCAA website. You do, however, need to be systematic.

MATHEMATICAL METHODS

Mary Papp and Allason McNamara. Mary and Allason are experienced Mathematical Methods assessors.

Do past exams

Pick an exam, work through it in the time allowed, then in correcting your work with the solutions, ask yourself would I get full marks or just some marks? Look carefully at the marks given to the question; ask yourself, 'for these marks what information (working out) do the assessors require?' In other words, try to think like the assessor. Make sure you read the Chief Assessor's report. This will give you guidance as to what was not handled well, and give advice on common errors that can be avoided. Once you have finished this put it aside, but do make sure you revisit this exam sometime in the future. You need to redo it in a shorter time and make sure that you can get it 100% correct! (Not just think you can – prove to yourself that you can!). It is better to do 20 exams thoroughly than 40 exams where you have not totally cleared away any or all misconceptions.

In working through past papers or practice exams, you have three main goals:

1. To revise the theory. Familiarise yourself with the style of question and the context in which the theory appears. Some analysis questions will combine two or more sections of the syllabus; for example, calculus can be combined with functions, algebra and probability.
2. To refine your 'supplementary notes'. These notes are only permitted with Exam 2. Are their certain calculations (examples) that you think would be of benefit? What about calculator short cuts? What does not appear on the formula sheet that you think is pertinent?
3. To hone your exam technique. Avoid common errors. Present your logic legibly and sequentially. Practise setting out!

General tips

- Use reading time wisely. Read through every question. Look at the marking scheme. Plan your strategy. In what order will you attempt questions? How much should be completed within 30 minutes?
- A question that is worth more than one mark must have accompanying reasoning.
- Be aware of key instructions. **Show that** effectively means prove. The answer is given, so the marks are awarded for methodology. **Hence** means that you must use a previous part of the question as supportive reasoning in this part of the problem. **Define the function** means give the rule and domain. Make sure you label your asymptotes with their equations or key points with their co-ordinates if asked to do so. If the question requires the answer to be written in a specified format, make sure answer in that format.
- As you complete a question get into the habit of checking that you have answered it specifically.
- Does the question ask for more than one thing; for example, the domain and range?
- Have you used correct notation? For example, if a question states a function say $f(x)$, and then asks for the inverse, the answer is not $y = \ldots$, but rather $f^{-1}(x) = \ldots$
- Is your answer reasonable? Consider restricted domains. Probabilities cannot be bigger than one or less than zero. For functions like $y = \log_e(f(x))$, $f(x) > 0$ or $y = \sqrt{f(x)}$, $f(x) \geq 0$.
- Sometimes a well labelled diagram is best; for example, drawing a tree diagram or a graph showing the area required for a normal probability distribution question.

Specifics for Exam 1

This is an exam worth 40 marks, a one hour time limit and no technology. Traditionally this exam focusses on the key concepts and will involve algebraic and arithmetic manipulation.

Things you need to practise include:

- Factorisation and expansion skills (including factor theorem).
- Transposition skills.
- Being careful with the placement of brackets; for example, $(2x-1)(3x-1) \neq 2x-1(3x-1)^2$.
- Legibility; for example, x should not look like pi, 's' should not look like a 5 and a 4 should not look like a 9 and indices should be obvious.
- Knowing your exact trigonometric values.
- Correct use of units.
- Avoiding poor expression (generally because you are in a hurry); for example, $\frac{1}{2x} \neq \frac{1}{2}x$ and 3^3 is not 9.
- Using negative signs correctly.
- Avoiding transcription errors.

Specifics for Exam 2

What applies to Exam 1 also applies to Exam 2, although, there are a few more things to consider.

- Do not spend too long on the multiple choice.
- Give exact answers unless otherwise specified.

- Make sure your calculator is in the correct mode.
- Use mathematical notation, not calculator syntax.
- Know how to reset your calculator if it is taking too long to generate an answer.
- Rounding should not occur until the final answer.
- Define functions where appropriate, to save time.
- For questions worth more than one mark, you must show some working. Often this means writing down the rule you are using and then the answer.

Remember the exams are an opportunity for you to show what you have learnt. The assessors want to see what you do know!

SPECIALIST MATHEMATICS

Sue Garner. Sue has been teaching senior mathematics at Ballarat Grammar for 25 years and is an experienced Specialist Maths assessor.

The changes for 2020 in Specialist Mathematics mean that the whole Area of Study: probability and statistics, has been deleted. This means that you can ignore questions from Area of Study 6 in the 2016-2019 past exams.

Do past exams

Make a decision on how many past exams you think you can manage to complete. Because of their style and content, past exams come in three sections 2006-2009, 2010-2015, 2016-2019. If you complete all of these exams this means there are two a year totalling 28 papers as well as the recent year's NHT papers.

It's good idea is to select a starting point and work forwards so that you reserve the most recent papers, 2019 Exams 1 and 2, to do for your last day or night's study session before the exams. Carefully read the examiners reports either after or during your practice of that paper. The most recent years will be the best ones to concentrate on. Many of you will have done the Maths Methods exams on the previous two days, so pace yourself well. Commercially written trial papers are also an excellent source of practice.

Specifics for Exam 1

Specialist Maths Exam 1 is often a relief where, if you know your material, you should be able to work through it. It's a great feeling to recognise every question type during reading time. Be alert to common mistakes:

- Not showing working steps in 2 and 3 mark questions; the assessors will not be impressed if you show you can do a 3 mark question in your head.
- 'Fudging' proofs where you get to the required ending with dodgy algebra.
- Making arithmetic mistakes.
- Assuming all questions in mechanics are using constant acceleration.
- Assuming all questions in kinematics are in terms of t.
- Trying to solve a DE when the question asks you to 'verify'.
- Assuming all integration can be solved using change of variables.
- Not labelling graphs with equations of asymptotes and axial intercepts.
- Finding an exact answer and then decimalising it when not specifically asked for, not even using $\approx$.
- Giving θ if the question asks for $\cos(\theta)$.

Be confident with your algebra and graphs and if you make a mistake, don't erase, but just cross out and do again in a space available. If this space isn't available make sure you draw the examiners' attention that you have attempted it perhaps on a spare or the last page. Sketch your graphs in pencil and watch your concavity by perhaps plotting another point to carefully sketch the shape. Label endpoints with coordinates.

There will be some difficult algebra and plenty of practise of past exams will only help this, but don't leave blank spaces! Some work is better than none, and you may even be correct enough for 1 mark.

Specifics for Exam 2

Specialist Maths Exam 2 will always be a difficult paper. The first decision is whether to attempt multiple choice section or the extended response section first. It is disappointing if you take too long on multiple choice and turn to the last long question and realise you know it well and have run out of time. Make this decision before your practice of at least the previous three years of exams. Allow 40 minutes (at most) for the multiple choice section, and expect some of these questions to require deep thought.

Some pointers for Exam 2

- Be aware when your calculator needs to be in complex form, decimal or exact form, radian or decimal
- Use unrounded decimals throughout the question even if you have had to round off in an earlier sub-section
- Do not write calculator syntax
- Do not write what you would have done if you had time, there are no marks for this!
- Make sure you label all forces in force diagrams, even if not asked for
- Answer every part of the question for example; don't leave off the y-value, the unit vector, the magnitude of a force, the constant of integration, the tildes, the dx
- Don't oversimplify expression when this is not required. Some students go on and on simplifying trig expressions when not required, making mistakes and losing marks on the way.

In preparation for Exam 2, refine your bound notes, but there is no need to rewrite them. As you go through your past papers, add into your notes any gaps in your knowledge, good examples of questions you have come across and reminders you think you will need. A summary and index at the front or back will help. Colour code corners of pages if that helps you. You may be surprised - you might not look at your notes as much as you expect.

The exams are written for you to show what knowledge you have, not to trick you, so go in ready to show how much you have learned this year. Good luck!

Further Mathematics is covered on page 8.

FURTHER MATHEMATICS

PREPARING FOR VCE EXAMS (CONT.)

Helen Haralambous, Mary Papp, Allason McNamara, Sue Garner and Fiona Latrobe

Fiona Latrobe. Fiona is an experienced Further Mathematics assessor.

Further Mathematics examinations this year will cover all material in the VCAA Study Design applicable from 2016 onwards, with the major change being that students will only be required to complete one module rather than two.

As in past years, students need an in-depth understanding of concepts and an ability to apply these, as well as an ability to use their CAS calculators selectively. Students should complete past examinations and trial examinations to time and carefully read the examination reports and compare these with their solutions. To ensure that as much revision as possible is completed, it is often useful to complete the examination to the end of the allowed time in one colour pen and, if some of the paper is incomplete, then finish this section in another colour so that you can monitor your progress.

The main change for students this year will be a change in percentages for each section of the exam. The data analysis section of the core increases from 40% of the exam to 50% of the exam, while the recursion and financial modelling section of the core increases from 20% to 25%, with a similar increase in their selected module. The increase in marks in each section means that students will be essentially examined on nearly all concepts with little room for knowledge gaps.

Examinations will remain at 90 minutes, with 15 minutes of reading time, so students should assign 45 minutes to the data analysis in each examination and 22.5 minutes to each of the recursion and financial modelling and their selected module.

Past VCAA examinations remain an important resource with no need to delete material from exams from 2016 onwards, but when completing these examinations students should be mindful of their timings, allowing only 12 minutes for reading and a total of 72 minutes only for the examinations (36 minutes for the Core and 18 minutes each for Recursion and Financial Modelling and their selected module).

Ideally, students should complete some trial examinations written to the Adjusted Study Design for 2020 so that they get the best sense of appropriate timing for a 90 minute examination with 15 minutes reading time.

Feel free to distribute these tips to your students. MAV wishes VCE students all the best for the 2020 exams.

ONE MINUTE WITH...ASHA RAO

I'M..

I'm Asha Rao, a Professor of Mathematics. I am the Associate Dean of Mathematical Sciences at RMIT University. I've just taken on another role as Interim Director of the Australian Mathematical Sciences Institute. I split my time across both roles.

ON MY DESK..

Right now, my desk is my dining table! I do a large amount of administration for both RMIT and AMSI which includes looking at how teaching will look like in 2021, and how we will organise seminars, workshops and summer school.

I'M READING...

I mainly read non-fiction as I find real life much stranger than fiction. I love reading about humans is all aspects. I love history and my reading list includes Bruce Pascoe's *Dark Emu*, and *Early India: From the Origins to AD 1300* by Romila Thapar. Another is the medical genre, including *The Emperor of Maladies* by Siddhartha Mukherjee and *Being Mortal* by Atul Gawande. I am reading *Delusions of Gender* by Cordelia Fine. This book discusses the changes (and non-changes) in gender and equity over the years. Another at my bedside is *Talking up to the White Woman: Indigenous Women and Feminism* by Aileen Moreton-Robinson

SOMETHING INTERESTING I'VE LEARNT THIS YEAR IS...

That teachers are amazing. When the need arose, we went online and have worked hard to give students as close an experience to face-to-face as possible. We have learnt a lot – the skills and technologies we have learnt over the last six months will stand us in good stead in the new normal that will come after COVID-19.

I'D LIKE MY FELLOW MATHEMATICS EDUCATORS TO KNOW...

Two things: the first is to carefully watch what we say to students regarding their maths ability. It is so easy for a student to fall into the trap of thinking they are not good at maths. We need to convey the fact that maths is hard but finding it hard doesn't mean they are no good at it. More than anything else, one can only become better at maths by practicing it – that is what makes it easy. But also, that easiness is only until the next concept. I have always said, diving has a degree of difficulty, so why not mathematics? Anyone who does diving knows they need to practice – so is the case with mathematics.

Secondly, relate maths to real life. Point out instances in real life that are examples for the maths the students are doing. As someone said, the students will come for the examples, but then stay for the maths.

Professor Asha Rao in her Penrose tiled jacket.

PC OR MAC?

I am definitely a PC person. I find the open source community is much stronger in the technologies that I use such as Latex, which is a compiling package for editing and publishing mathematics.

DURING COVID-19 LOCKDOWN I'VE BEEN...

Busy with adjusting to online teaching and finding new ways to connect with my students – using little things to give them the feeling that even though we are doing the learning via technology, I am here for them, as much as I was when it was face-to-face. On weekends, I like to sew and find ways to use mathematics in my sewing. You can also find me in the garden or on a walk. After lockdown ends, I'm looking forward to a long walk in the Lerderderg Forest.

I GET INSPIRATION FROM...

The people around me. Each is coping with COVID19 in their own way, and many are finding it difficult. However, they are still taking the time to offer support to others. The resilience and generosity is amazing and makes me feel fortunate.

MATHEMATICS CAN...

Open doors that you did not even think existed. The future of the world hinges on mathematics – embrace it and it will lift you up. (Okay that's two sentences!)

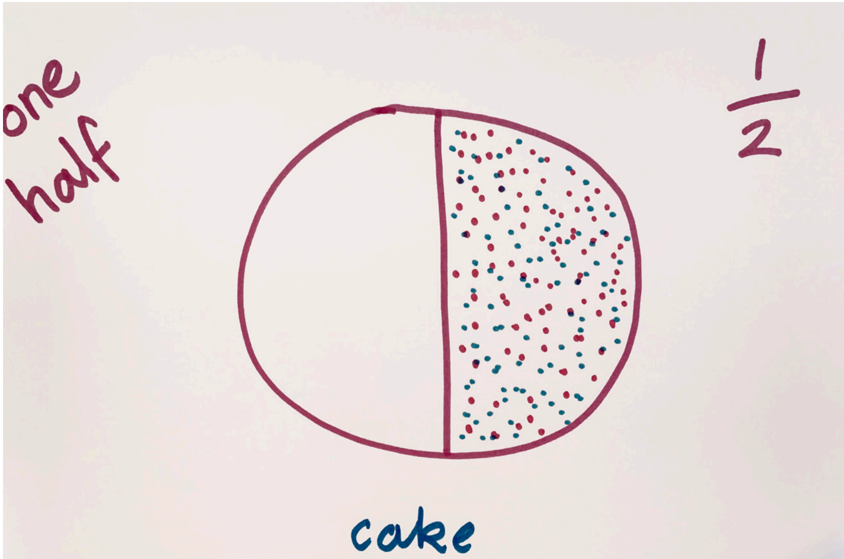
Who would you like to see profiled in *Common Denominator*? Email suggestions to office@mav.vic.edu.au.

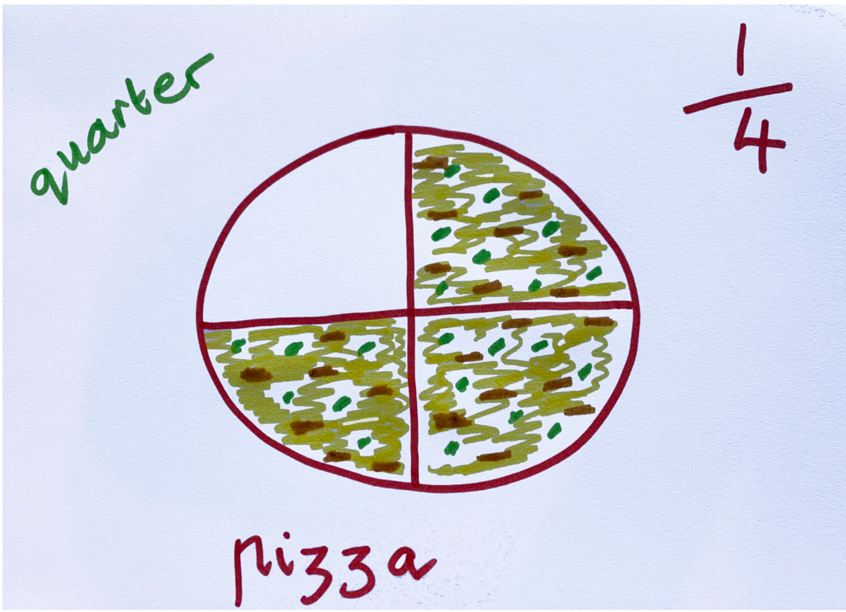
UNDERSTANDING FRACTIONS

Lanella Sweet - Extension and enrichment teacher, Wesley College

When introducing primary aged students to fractions, they are keen to jump straight into the idea of writing down the abstract form. Many students would have experienced real life situations with the use of fractions, and often have the pre-conceived idea that it is more important to demonstrate fractions on paper in the abstract form. This can sometimes demonstrate misunderstandings or lack of foundational knowledge, which results in gaps in the student's fraction understanding. Often students are curious and motivated to work in the abstract form, sometimes at the disconnect of the broader fundamental concept being explored.

The natural excitement of young students exploring fractions is often tangible! The curiosity and enthusiasm of students presenting their thoughts in this manner is important to encourage, however it is necessary for the learners to fully understand the fundamentals of properties, patterns and real world connections for application and generalisations to take place successfully. It is important to balance the student's sense of excitement when exploring fractions with learning appropriate fundamental skills in the primary years. It is essential to keep the student's interest and confidence aligned, as they progress through the learning of fractions basic concepts to application of such knowledge.





USING CONCRETE MATERIALS

Using concrete materials throughout these formative years is often a reminder of the bigger picture of mathematics and allows students to bring back their thinking to visual, logical representation. This thinking coupled with linking mathematical concepts to real-life scenarios is advantageous. The benefit of using concrete materials along with the motor sensory experience can enhance students initial understanding of fraction concepts. Sowell (1989) and von Glaserfeld (2002) have argued for a deeper understanding of mathematical concepts to be learnt through the continuing engagement of students manipulating concrete materials. In the early years, concrete materials are often used to describe, represent, and compare fractions. In the later years of primary school this can be extended to demonstrate/build knowledge of recognising, interpreting, and using formal fraction symbolism and notation to solve problems.

EVERYDAY LIFE AND REAL LIFE CONNECTIONS

When opening discussions about viewing fractions in real life, often students combine their understanding of fractions with the experiences of cooking and shopping. Cooking and using recipes are easily linked with the visual and manipulative task of seeing and using various cooking materials, such as cups and spoons to show the parts of a whole. Many young children have experienced shared cooking experiences with family and might often have simple initial prior knowledge of using one cup and half a cup of ingredients.

Financial maths and the understanding of the money system in Australia, is naturally linked with fraction knowledge too. More often students have experienced the concept of 100 cents equal to one dollar and therefore half of one hundred is represented as 50 cents. It is interesting to note these students who can transfer their knowledge of half one dollar and one full dollar equivalence, often do this as a natural progression, when thinking about connecting their prior knowledge of money to understandings about fraction representation. For example, students expressed 25 as being one quarter of 100, therefore 25 cents can be thought as being equal to one quarter of 100. Consequently, connections of equivalent fractions were made from linking 25/100 (thinking in the notion of money) to one quarter (thinking in terms of fractions). Such students who display solid understandings of financial maths, due to simply the exposure and experience that happens within their everyday life could make further connections and understandings of fractions.

Context examples of fraction application in everyday life experiences:

- time concepts
- drawing
- navigating / locating
- measurement
- planning
- organisation charts and allocation concepts
- travel planning and budgeting
- building ideas and planning
- designing .

These concepts from everyday life can be further examined within integral parts of the mathematics curriculum such as:

- chance
- data representation and interpretation
- extension of number line
- statistics
- probability
- geometry
- rounding
- estimation
- technology use
- reasoning
- percentages
- decimals.

Key ideas about fractions that need to be covered:

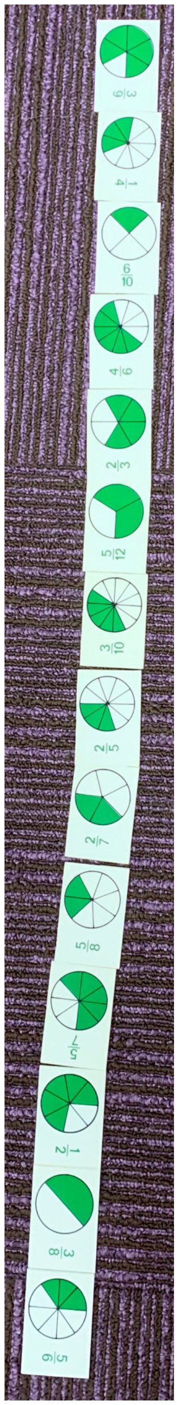
- recognise and describe fractions
- interpret and compare fractions
- model and represent fractions
- count by fractions at various intervals
- understand place value extension – tenths, hundredths, etc
- convert and recognise equivalent fractions
- understand decimals and compare and order with fractions
- understand the relationship of numerator and denominator and
- convert mixed numbers and improper fractions
- perform all operations with fractions
- convert and compare fractions, decimals, percentages
- terminating decimals and recurring decimals
- apply knowledge and make connections to solve complex problems.

VISUAL REPRESENTATIONS

Students who make visual connections initially can represent, understand pictorial fraction and other visual tools with ease. Engaging students to think about fractions from this perspective leads them to represent objects that are meaningful to their life, such as cakes or pizzas. These representations of circular designs are easy to work with as they can partition into easily recognisable pieces to visually represent fractions. The commonly used circular representations should be extended to include other regular polygons. By students using drawn, designed, and previously understood comprehension of visual representation, they can successfully engage in describing simple and equivalent fractions. They can therefore understand patterns and logic as a base to all fractions being part of a whole when combining real life drawings, e.g. cakes and other shapes, e.g. rectangles or squares. When learners have this type of inter-connected knowledge, they can make strong mental connections and relationships with the information, use it, and apply this to appropriate fraction understandings. These are useful tools and are seen to have benefits of comprehending and transferring of interrelated skills and strong mental structure, within mathematics. (Wilensky 1991)

FURTHER LEARNING AFTER THE FUNDAMENTALS OF FRACTIONS

Once a solid understanding of fractions has occurred with pictorial representation, meaning and relationship with the real world, along with writing, comprehending and using manipulatives to order, demonstrate, model, compare and represent fractions, students will have the fundamental skills to extend their knowledge further and gain success with this area. Some examples of these concepts will involve the ability to find equivalent fractions, connections to decimals, understanding percentages, problem solving with fractions, representing data, percentages, and decimal numbers, and manipulating denominators and numerators appropriately. Developing understandings to further transfer knowledge will then take place, which will allow the student to excitedly establish themselves as successful learners of fractions!



REFERENCES

Sowell, Evelyn J. (1989) Effects of Manipulative Materials in Mathematics Instruction. *Journal for Research in Mathematics Education, 20*, 498-505.

Von Glasersfeld, E. (1992). Sensory Experience, Abstraction, and Teaching. In L. Steffe & J. Gale (Eds.), *Constructivism in Education*, (pp. 369-385). Hillsdale, N.J. Erlbaum Rosemaree Caswell.

Wilensky, Uri. (1991) Abstract Mediations on the Concrete and Concrete Implications for Mathematics Education. *In Constructionism*, edited by I. Harel and S. Papert, 193-99. Norwood, N.J.: Ablex Publishing Corporation.

Jigyel, K., & Afamasaga-Fuata'i, K. (2007). Students' Conceptions of Models of Fractions and Equivalence. *Australian Mathematics Teacher*, *63*(4), 17-25.

STIMULATING THINKING

Jennifer Bowden – Education consultant, Mathematical Association of Victoria

A picture sparks 1000 maths concepts! Use this picture as a prompt to stimulate thinking. If you have other ideas for investigations or lessons that could stem from the ideas here, add them to the conversation on our social channels. You can find us on Facebook, @mathematicalassociationofvictoria and on Twitter, @mav_info.

FOUNDATION - YEAR 2

- Draw your ideal zoo. Which animals did you include? How many of each kind? Which animals will live together and which need to be kept apart?
- What's your favourite animal? What do you like about it? Name five features that make your animal special.
- Write down 10 different animals:
  - Order the animals from smallest to largest. Is there another way you can order your list?
  - Survey your friends and family to find out which are their top three favourites. Graph the results.
  - Research the top three favourite animals. For each of the animals, find out how tall they are, how fast or slow they move and how long they usually live for.
- The Aldabra tortoise can live for 120 years. If you met an Aldabra tortoise that was 110, what year was it born?
- There are lots of ways to make origami animals. Look up instructions online to create an animal using origami. Once you are finished, look at your animal and see if you can name shapes in your paper folding.
- Download a tangram template online. Using the pieces of the tangram, can you create an animal?
- Orang-utans have an arm span of about 2.2 metres. Could they complete the monkey bars at your school in just one swing? How could you work it out?
- There are many different leaves in the image. Can you find a collection of leaves and create a pattern that repeats? If you can't find leaves you could draw them.

YEARS 3 AND 4

- Look at the animals in the picture. Think and record three ways you could sort or order the animals.
- Many plants have symmetry. Take a photo or draw five plants in your home or local environment that have symmetry.
- Write down the names of five different animals:
  - What do they have in common? What's different about them? Can you create a chart to show your thinking?
  - Research your animals and find out where they live, chart the results on a map of the world.
  - Count the legs of each of the five animals you've chosen. How many different ways could you make 80 legs? Would you need two of a particular animal? Or maybe more?
- Go down to your local park or walk your neighbourhood, what animals do you see? Keep a tally and record your results on a bar graph.
- Using LEGO or other materials in your house, build your favourite animal.
- Design your ultimate zoo. On an A4 grided piece of paper, draw animal enclosures, food outlets, bathrooms, paths, entrance, gift shop and anything else you'd like to include. Write down the co-ordinates of seven different destinations.
- Find a map of a zoo, look at the scale of the map. How long would it take you to walk to see your favourite animal?
- Elephants poo a lot! About 12 times each day. On average, an elephant produces 100kg of poo each day. How much is that each year? Can you covert the amount to tonnes?

YEARS 5 AND BEYOND

- Create a zoo in Minecraft.
- Animals reproduce at different rates. Elephants gestate for almost two years, flamingos gestate for about a month. Choose six animals and research their gestation periods. If those six animals lived on an island, how long would it take for there to be 100 animals on the island (assume that the animal can magically reproduce)? What's the shortest possible time? What's the longest time?
- The cheetah is the fastest land animal, it can run at speeds of about 100km/h, if a cheetah can run the length of a field in 14.2 seconds, what is the length of the field?
- The peregrine falcon is the fastest aerial animal. If the falcon and the cheetah were in a 60km race, how long would it take each animal to finish the race? You might need to research the speed of a falcon.
- Look at the picture and estimate the height of a flamingo and a toucan. Validate your result by researching on the internet. Using the results, how does the height of both birds compare to your height?
- The Amazon rainforest has an abundance of plants and animals. How far is the Amazon from your house? If you could fly at the speed of a peregrine falcon, how long would it take you to get there? What if you could crawl at the speed of a Aldabra tortoise, how long would it take you?
- A single cow can produce nearly 200,000 glasses of milk in a lifetime. Assuming that a glass is 250ml, how many litres can one cow produce? How many cows would it take to produce one glass of milk for each person in Australia?

VCE SACS: NETWORKS

Andrew Stewart

The good aspect of SACs is that there is not only more time available to test student understanding of key topics, but also time to explore topics that are not always featured in examinations due to time constraints. In this article, I will explore two of these in more detail.

HUNGARIAN ALGORITHM

This algorithm is ideal for use in a SAC since there is time available to use the algorithm in its entirety, unlike the time constraints imposed by an examination. Examination 2 examples are often quite simple, or students have been given key information/processing results to assist them. Recent examples of these approaches can be found in MAV's Trial Exams (2017: E2 Q2 and 2018: E1 Q6).

Good sources for SAC problems are business-maths textbooks, from the bargain table at good bookshops. I have listed two useful sources in the references.

The example (above right) uses data modified from Gallagher & Watson (1985) within the context of the original problem.

Modified from 2017, Exam 2 Question 2

Alf, Barbara, Clarice and Donald all work at the VicRoads office. They each serve at the counter and can process registrations (R), licences (L), learner's permits (P) and roadworthiness certificates (W). The average times in minutes for each task for each person are shown below in a table:

	Registrations (R)	Licences (L)	Learner's permits (P)	Roadworthiness certificates (W)
Alf	14	17	17	12
Barbara	15	18	17	13
Clarice	15	19	20	16
Donald	17	14	16	17

The allocation of one job per person is required to ensure that the minimum time is taken on tasks.

For this task, students must complete the minimum allocation using the Hungarian algorithm, showing all working. Providing table templates may reduce the total time required for completion.

This problem has been designed so that there are two quite different ways to draw the minimum number of lines to cover all the zeros in the first instance. Students should explore completing the solution based on each of these situations. Did one way of drawing the lines make it easier to complete the allocation than the other?

HAMILTONIAN CYCLES

Questions involving Hamilton cycles are fairly short and often just in the first examination or as an introductory question in the second. Some recent examples can be found in MAV's Trial Exams (2016: E1 Q3, 2018: E1 Q4, Q5, 2016: E2 Q1(b), 2017: E2 Q1(b) and 2018, E2 Q1(c)).

The most famous application of Hamiltonian cycles is the travelling salesman problem. This classic problem gained prominence in the 1950s when companies tried to assist their salesforce to visit all the sites in their area and return to their base travelling the least distance, or with the least cost.

For small numbers of vertices, a 'brute force' approach of writing down every possible cycle will find the shortest one. With four vertices, there are 24 possible cycles, but only three of these are unique. All the others are either the reverse of a unique cycle, or starting at another vertex on a unique cycle. (See Figure 1. *ABCDA* is essentially the same path as *BCDAB* as *CDABC* as *BADCB* and the same in reverse: *ADCBA*, etc.) Thus there will only be a maximum of three different total lengths. For five vertices there are 120 possible cycles with just 12 unique cycles, and for six vertices there are 720 possible cycles, with just 60 unique cycles. For larger numbers of vertices a more efficient method is needed to find the shortest path!

One frequently used method is the nearest-neighbour algorithm – a general name for the algorithm that lies at the heart of both Prim's and Dijkstra's algorithms. From a particular starting vertex, choose the shortest distance path to another vertex, then choose the shortest distance to a different vertex and, after visiting all the other vertices via the shortest available distance finally returning to the starting vertex. A good strategy would be to start at one end of the shortest side, and then compare the value obtained for starting at the other end of this side. These values could be compared to starting at either end of the second or third shortest sides.

THE FOUR TOWN PROBLEM

A salesperson has to visit four towns, starting and finishing at one town. Figure 1 shows the road distances in kilometres between the towns (labelled *A*, *B*, *C* and *D*).

The task is to find the town from which the shortest overall journey starts (and finishes), and to determine in which order each of the towns should be visited. Students should compare their findings for the three unique cycles from the 'brute force' approach to the solution with the effectiveness of the nearest-neighbour algorithm in finding the shortest cycle.

In this case, none of the four nearest-neighbour algorithm attempts will yield the shortest cycle. (There will be two pairs of identical cycles, neither of which is the shortest distance.) Students will need to identify and evaluate the three unique cycles to determine the minimum total distance.

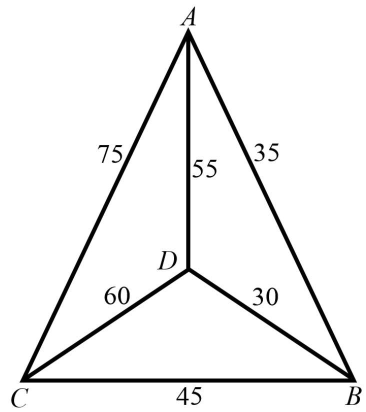


Figure 1. The four town problem.

However, if one or more distances between towns is changed, then the shortest cycle distance can be found by both brute force and nearest-neighbour. From the diagram above, this can be achieved by reducing *AC* to 65 km or to 50 km, or lengthening *BD* to 40 km.

To make this task more challenging, give the students the table with the distances (shown below), ask them to construct a diagram and then find the required distances.

Edge	*Km*
AB	*35*
AC	*75*
AD	*55*
BC	*45*
BD	*30*
CD	*60*

THE FOUR TOWN ALGORITHM

Because there are only three distinct routes, there is a very simple algorithm (method) for finding the shortest distance for four towns. Each route uses four edges and for each route the edges left out are either *AD* and *CB*, *BD* and *AC*, or *CD* and *AB*. These each consist of one 'middle' edge and the opposite outside edge. Find which of these has the largest total, and the shortest path will contain the other four edges. Refer to Figure 1.

THE FIVE TOWN PROBLEM

Figure 2 shows the distances between five towns (labelled *A*, *B*, *C*, *D* and *E*), in kilometres. Find the town from which the shortest overall journey starts (and finishes), and determine in which order each of the towns should be visited. In this case, students should use the nearest-neighbour algorithm to find the shortest cycle.

When starting from any vertex, there will be only one cycle that the nearest-neighbour algorithm will generate. The key difference between this and the previous problem is that application of the algorithm is very much dependent on the starting point, and thus trying to start anywhere on the shortest cycle may not necessarily continue due to the application of the algorithm or the sites left to visit.

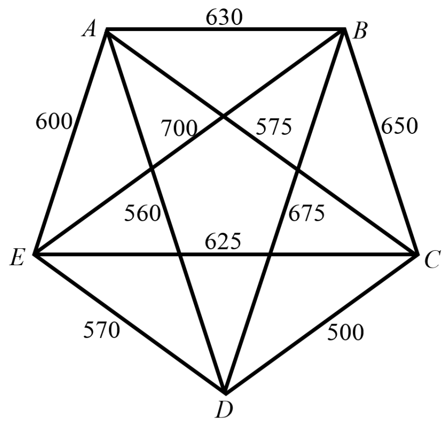


Figure 2.

The distances between the towns are given in the table below for clarity.

Edge	*Km*
AB	*630*
AC	*575*
AD	*560*
AE	*600*
BC	*650*
BD	*675*
BE	*700*
CD	*500*
CE	*625*
DE	*570*

As in the previous problem, the shortest cycle will not be found by the nearest-neighbour algorithm for this data. It's not just about selecting the shorter lengths. You must also avoid the longer ones. Sometimes starting with the shorter lengths forces the long distances to be included. Given there are just 12 unique cycles (each with a total length different to the others), it would be a reasonable SAC activity to find them all – students could write down the cycles commencing from a particular (given) vertex.

However, if one or more distances between towns is changed, then the shortest cycle can be found by the nearest-neighbour algorithm. From the data provided, this can be achieved by reducing *DE* to 470 km or reducing *BE* to 660 km or reducing *AE* to 540 km or lengthening AC to 660 km. The value could be confirmed by students checking the unique cycles commencing from a particular vertex.

SPECIAL NOTE

The nearest-neighbour algorithm values for a cycle length can differ from the shortest journey value by up to 25%. I have included it here for practice in its use, and to show that a variety of techniques can be used to solve interesting problems.

REFERENCES

Consortium for Mathematics and Its Applications (1991). *For All Practical Purposes – Introduction to Contemporary Mathematics*. W. H. Freeman (New York).

Gallagher, C.A., & Watson, H.J. (1985). *Quantitative Methods for Business Decisions*. McGraw-Hill (Singapore).

The Travelling Salesman Problem https://en.wikipedia.org/wiki/Travelling_salesman_problem

MTQ 2020

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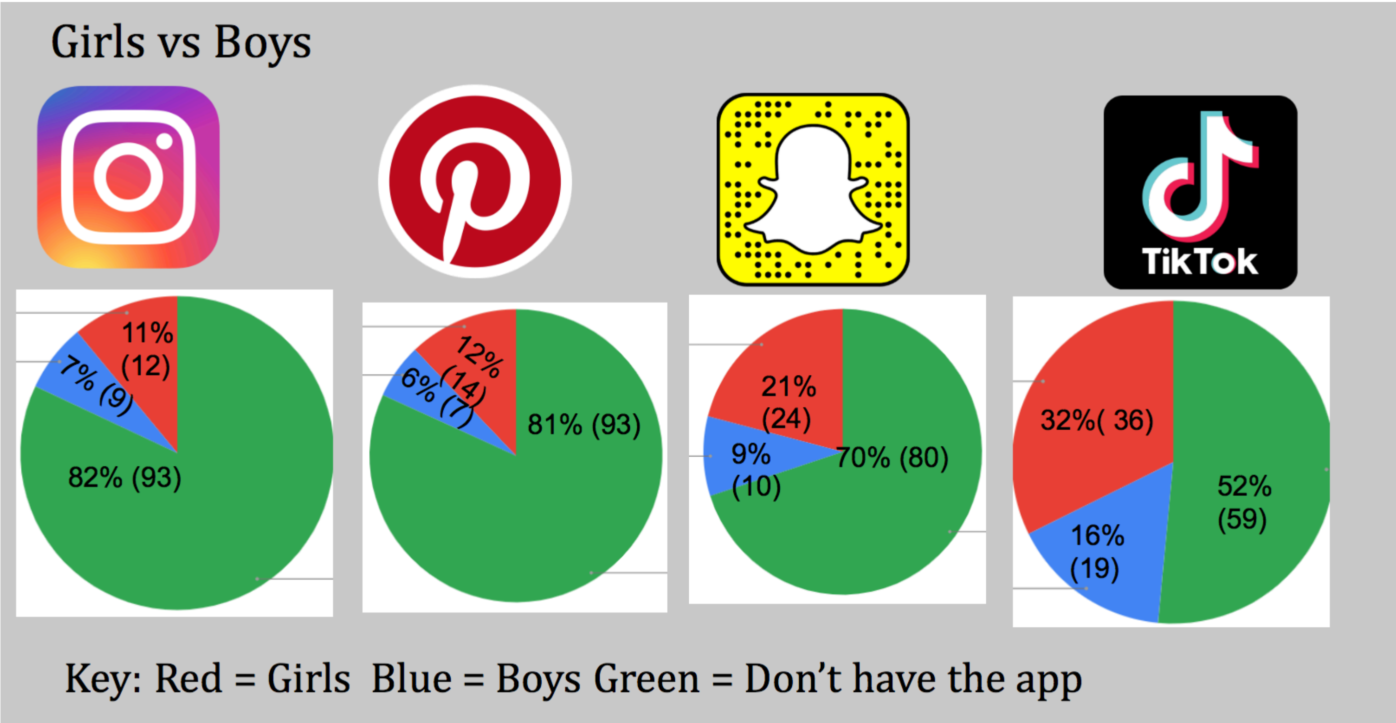

How much time does a 9-12 year old use social media? These graphs are from an MTQ investigation by Year 5 students Liliana and Scarlett.

2020 has certainly provided its educational opportunities and challenges. The prospect of running the Maths Talent Quest in its usual format wasn't an option due to COVID-19 restrictions. The team at MAV re-assessed our goals and created a unique opportunity for students to complete investigations and enter the competition in an innovative, online environment.

The Maths Talent Quest provides students with an opportunity to apply their mathematical skills to real life mathematical examples. The prospect of exploring, collaborating, investigating and sharing findings in a personal and unique method is a challenge within an online learning environment. Nevertheless, many schools decided to meet this challenge and provide a platform for enquiry for their students.

In 2020 the MAV provided two platforms for MTQ, school-based judging where schools were provided with resources, advice and certificates and the traditional state-wide judging with modifications to judging process.

COVID-19 restrictions meant that our traditional method for of judging MTQ investigations in a face-to-face setting with the physical investigations would not be possible. Fortunately, MAV had moved our judging to a digital platform in 2019 enabling judges to assess investigations from their own homes or schools.

Students worked as individuals, small groups and classes over Term 2 to complete their investigations. If you can cast your mind back, Term 2 was quite disruptive - students, teachers and families had to be very flexible and adaptive. It is a credit to the students that they were able to rise to the challenge and continue to work mathematically to produce outstanding investigations.

Registrations and judging closed in August with over 180 entries submitted from 48 schools across Victoria, three schools from Tasmania and the Victoria International School of Sharjah.

We were extremely proud of the educators and students who showed resilience and a spirit of challenge in mathematics to complete their investigations. Teachers were agile in learning a variety of new platforms and digital skills to be able to register and upload students work.

A digital platform encouraged new schools, especially those from a regional environment participate in the State-wide Maths Talent Quest. Students were innovative in the ways they presented their work, using a variety of traditional and digital means to present their learnings.

The quality of entries was of an extremely high level and competition for entry into the national level was strong. Most entries received distinctions or high distinctions displaying a level of learning that exceeded the expectations of students learning levels. Both Victoria and Tasmania were able to provide a strong quality and quantity of investigations across categories into National Judging hosted by the Mathematical Association of Western Australia, with virtual judging by judges across Australia. The results of the state-wide and national MTQ will be announced through a virtual ceremony in Term 4.

Like many schools, the MAV has pivoted and been agile - this has resulted in growing and evolving the Maths Talent Quest to incorporate a flexible online platform. The 2021 MTQ will incorporate a virtual platform allowing innovative digital investigations and stronger opportunities for our regional schools.

Many thanks to the students, teachers and judges for their work to make MAV's 2020 MTQ such a success. Thank you to our sponsors, La Trobe University and Casio Calculators Australia for their support.

COMBINATORIAL GAMES

Terence Mills

Most of us enjoy playing games, and we have done so since childhood. Games of strategy are particularly popular among mathematical folk.

There are mathematical games known as combinatorial games. Combinatorial games are usually two-person games, that do not involve chance, and have no hidden information. Noughts-and-crosses and chess are two examples. These games do not involve chance or randomness such as throwing a die or tossing a coin; all the information about the present state of play can be seen by both players; full disclosure. Combinatorial games are useful for engaging students in mathematical activities (Posamentier & Krulik, 2017).

Just this year, two remarkable mathematicians who have contributed greatly to combinatorial games have died: Richard Guy (1916-2020) and John Conway (1937-2020). Their ideas are important in the history of contemporary mathematics.

Richard Guy was an expert in number theory, graphs and networks, combinatorial games, and recreational mathematics. He wrote many papers and, amazingly, he kept his research until he was over 100 years old. As an Emeritus Professor, Guy continued to come to work almost until the end of his life. His final book (Brown & Guy, 2020) was published in 2020. In addition to his passion for mathematics, Richard Guy was a keen hiker and active in environmental and peace issues, and well-known in chess circles for his beautiful chess problems.

John Conway was truly an outstanding mathematician. The newspaper article by Roberts (2015, July 28) gives a good overview of John Conway and his work. The book by Roberts (2015) is a full-length biography.

Guy and Conway (1996) also wrote a fascinating book on numbers ... all sorts of numbers; integers, fractions, real numbers, complex numbers, numbers and nature, numbers and geometry, old chestnuts, unsolved problems — all in about 300 pages and written in an engaging style. Even the title (*The book of numbers*) hints at the authors' humour.

The book is replete with gems that would be useful in a mathematics class. For example, there is a gem in chapter 6 on fractions. The authors discuss Farey fractions (or Farey sequences).

Playing with Farey sequences will give all students the experience of placing fractions in order. After some experience, some curious students might frame their own questions about these sequences of fractions. The art of asking questions is very important in mathematical investigations.

The book of numbers could be enjoyed by an enthusiastic high school student, a university student, a mathematics teacher, or any mathematical person.

REFERENCES

Brown, E., & Guy, R. (2020). *The unity of combinatorics*. Providence, RI: American Mathematical Society.

Guy, R., & Conway, J. H. (1996). *The book of numbers*. New York, NY: Springer.

Posamentier, A.S., & Krulik, S. (2017). *Strategy games to enhance problem-solving ability in mathematics*. Singapore: World Scientific.

Roberts, S. (2015, July 28). John Horton Conway: the world's most charismatic mathematician. *The Guardian*. Retrieved from https://www.theguardian.com/science/2015/jul/23/john-horton-conway-the-most-charismatic-mathematician-in-the-world

Roberts, S. (2015). *Genius at play: The curious mind of John Horton Conway*. New York, NY: Bloomsbury.

MEMBER NEWS

MAV IS NOW A CHARITY

In February 2020 the MAV Board voted in favour, to apply for MAV to become a charity. This was after considerable work, investigation, research and legal advice taking place for a period of 18 months prior.

The day to day running of the association remains the same, and members and educators will not notice a different in products, membership or services offered. Many teacher associations are charities so MAV is not alone, and we look forward to MAV members ongoing support.

MAV will now report to the Australian Charities and Not-for-profitsCommission, www.acnc.gov.au.

WHY A CHARITY?

There are various benefits to MAV becoming a charity which were the basis of this Board decision. Key reasons include:

- MAV was previously self-assessed as Income Tax Exempt under certain criteria with the ATO. This carried potential risk and becoming a charity means MAV is Income Tax Exempt without this ongoing risk.
- GST Concessions and Fringe Benefit Tax rebates.
- Various cost reduction opportunities with charity-based discounts from service suppliers, for example MAV has significantly reduced monthly Microsoft licensing costs.
- Possible future application for DGR (Deductable Gift Recipient) status to set up scholarship fund for studying accredited courses in education.

MAV's current constitution has various clauses stating how governance is managed as charity, and MAV's operations will continue smoothly from this point.

If you have any questions about this change please contact Peter Saffin, CEO via email at psaffin@mav.vic.edu.au.

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WHICH ONE DOESN'T BELONG?

Alicia Clarke –School maths leader, St. Mary's Primary School

Which One Doesn't Belong? is a unique mathematical children's book. The book isn't a picture story book and doesn't have a storyline with characters or settings, rather it is a book to promote mathematical thinking, with mathematical problems presented in a book form.

The problems presented all involve shapes, and asks students to identify the shape that doesn't belong and provide a reason for their answer. As stated on the very first page 'Every page asks the same question, and every answer can be correct'. The first half of the book is an introduction, with four shapes shown and the question 'Which one doesn't belong? Why?' The following pages give examples of which one might not belong and the reason that may be the case.

The beauty is that it could be any of the four shapes, all for different reasons. This helps builds student's confidence – any answer can be the right one! These introductory pages also introduce students to vocabulary such as 'sides', 'angles,' as well as shape names. Students can then use this vocabulary when solving the problems further on in the book.

The remainder of the book presents four different shapes on each page, and always asks 'Which one doesn't belong?' These pages are great springboards for discussion, and could be done as a whole class, small group or even individual activity. While this book uses shapes and promotes the use of mathematical language to describe and compare these shapes, which fits within the strand of measurement and geometry, it also heavily promotes the proficiencies of reasoning. The Victorian Curriculum states that students are reasoning when they:

- Explain their thinking
- Deduce and justify strategies used and conclusions reached
- Prove that something is true or false
- Compare and contrast related ideas and explain their choices.

Which One Doesn't Belong? requires students to do all of these things! Here is one example of a session which could be implemented into a maths classroom using this book as a springboard.

The session was conducted with a class of Foundation students.

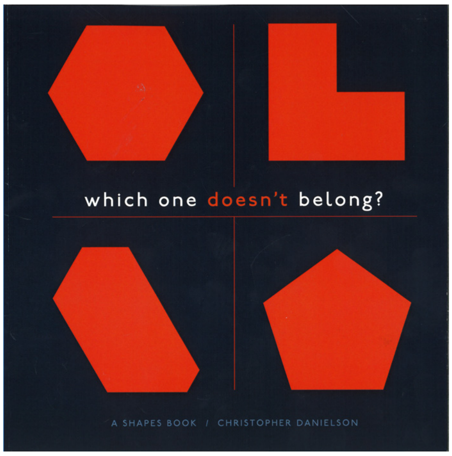
Victorian Curriculum Link: Sort and classify familiar objects and explain the basis for these classifications.

As well as addressing the above curriculum content descriptor, the lesson also address the proficiency of reasoning as described above.

To begin, we read the book as a whole class. While reading the first few pages, we discussed which one the book showed us didn't belong on each page and why. As a teacher, I emphasised words such as 'sides' and 'angles' as reasons for the shapes being the same or different. When we came to the pages which asked students to decide which one didn't belong, we had some great discussions. Students began using more vocabulary, such as round, straight, corners, points, curved and so on. We listed this vocabulary on the board.

After working through several examples as a class, we broke up into small groups, with each group taking a photocopy of one page of the book with them. They had to discuss as a group which one didn't belong on their page, and be able to give reasons for their answer. The challenge was to agree as a group – and from this challenge came the use of rich vocabulary and students reasoning that their choice was the best choice. Each group presented their page and their choice to the class.

For the next activity, students worked individually. They were given four shapes (concrete materials) and asked to think



about which one didn't belong and then present it to the class. Using concrete materials adds and extra element to the task as students can touch the shapes. You could also extend this to include 3D objects. In a subsequent session, this activity was repeated and each student was recorded justifying their choice as assessment for both shape and reasoning.

The above session is just one idea of how *Which One Doesn't Belong?* could be used in a classroom, and highlights how it could be used with whole class groups, small groups and individual students. It is often difficult to find resources that highlight the proficiency of reasoning so well, which makes this book a useful resource for all mathematics teachers.

This book is available from MAV's online shop, www.mav.vic.edu.au/shop.

VCAL: REAL WORLD MATHS IN STEM

Marco Nicolazzo – Peter Lalor Secondary College

As an engineering student studying in the late 1990's and early 2000's, the term STEM (Science, Technology, Engineering, Maths) was not an acronym well known in the industry. In my years as a student, if one intended to pursue a career in science, they would study a science degree. The same for technology, engineering and maths students. It wasn't as though each discipline was mutually exclusive. It was almost implied that an engineering student required competent knowledge of mathematics and technologies, just to mention a few.

It is great to see students engaged in applied learning. I am an advocate of real world learning in the classroom, as I believe this promotes the greatest opportunity to teach complex numeracy, as an example. However the tasks performed by these students were no different to tasks I completed in primary school. Using cardboard, tape and scissors shouldn't be the new norm to promote STEM. Are we just reprinting old craft lessons and rebadging them as STEM? When I was recently tasked with developing an engineering wing as part of the school applied learning strategy, I was adamant that the key principles of STEM were to be taught in an environment that also offered students improved employability skills.

I facilitated a cohort who were able to demonstrate exceptional maturity when tasked with hands-on learning. The cohort were most engaged when they were making something of purpose. My key task was to improve their numeracy skills. One of my first lessons was to teach triangulation through trigonometry. Conventional teachings would have me standing in front of the classroom, drawing triangles on the board trying to engage the students into understanding the sine and cosine rule. I always comprehended more quickly when I was forced to complete a hands on task that incorporated the key learning points. I even learnt faster after making mistakes. With that in mind, I adopted a similar strategy.

Our arts department was in need of new easels, it was agreed that my cohort would construct them. An easel is made up on many different sized triangles, so this would be the perfect opportunity to teach students trigonometry in a hands-on environment. Having students produce a tangible object enabled me to improve

student engagement. Rather than having students complete a workbook on angles and calculating hypotenuse, now they could make a full scale easel. The task was made even more challenging when the project was student led. Presented with some basic parameters, students needed to engineer a design and calculate cut sizes.

This is where the true fruits of a well-constructed STEM activity can demonstrate comprehension of key concepts while promoting employability skills. With material limitations, students were forced to develop a design that was cost effective to manufacture, as well as meeting the needs of the client. The key learning point was to ensure students were able to calculate the length of the hypotenuse. One of the first challenges of teaching trigonometry and triangulations is that hypotenuse does not always equate to height. The ability to demonstrate, by presenting students with completed works, allowed the cohort an opportunity to evaluate and analyse the learnings from various angles. Being able to view mathematical theories by walking around an object allows a greater connection between classroom teachings and real world applications.

The task gave students a tangible understanding of the importance of angles when constructing objects made up of multiple triangles. Students were able to visualise angles in person, develop construction and manufacturing skills that could be adapted in the workforce. A visual memory record has now been formed that will be present with these students for years to come. This visual record is far more beneficial and effective than any worksheet or teacher presentation, as the event is now stored in the students long term memory. The hands-on environment promoted a culture where students asked questions. It prompted a need for teams to work together to overcome (engineer) a solution. It also promoted an improved relationship between student and the teacher.

STEM is a great concept. A means to incorporate a range of disciplines in the single task. The challenge for facilitators is to ensure that students are able to walk away with a skill that the student will remember for a good portion of the rest of their lives. STEM should not be a lesson where students 'make things', or move the cursor and click print. There is no key learning in watching a printer spend the next few hours printing a project that students constructed on a computer program by moving a few shapes. STEM should be about allowing students a hands on experience to comprehend key learning in an environment that promotes confidence, independence, and employability skills.

BOOK REVIEW

Carmel Delahunty – Mathematics education consultant

PRIMARY AND MIDDLE YEARS MATHEMATICS: TEACHING DEVELOPMENTALLY - FIRST AUSTRALIAN EDITION

Pre-service teachers, in-service teachers, mathematics leaders, and consultants who are familiar with Van de Walle and colleagues' 'teaching developmentally' texts will welcome this first Australian edition. In this edition, authors Van de Walle (posthumously), Karp and Bay-Williams are joined by Amy Brass, as well as contributors from Australia, (see reference).

The overarching focus of the text is the exploration of teaching mathematics through problem-solving. This choice is to satisfy 21st Century skills including 'critical thinking, communication, collaboration, creativity and [the utilisation of] technology. (Van de Walle et al., 2019, p. 34).

The text is divided into two sections: the first is concerned with important ideas in mathematics learning and teaching, such as equity, and the second section is devoted to mathematical content for students in Foundation to Year 9.

SECTION 1

You can dive into this book from various points. The first section has foundational information: what is means to teach, know and do mathematics in the 21st century, how to plan for teaching problem-based mathematics, and how to create assessments for learning.

Recently, requiring information about mathematical vocabulary and EAL/D students, I first went to the 'brief contents' section at the front of the text and saw immediately that I needed to access Chapter 6 *Teaching Mathematics Equitably to All Children*. From this starting point, I found the information I needed, including tables and figures; access to the teacher resource pages is granted after establishing an account with Pearson, the publisher.

I also used the index to search for information on classroom discourse, and was directed to Chapter 3 Teaching through problem-solving (Van de Walle et al., 2019, pp 51 – 55). Included herein is a brief discussion of Smith and Stein's (2011) 5 practices for orchestrating productive mathematics discussions, and Chapin, O'Connor and Anderson's (2009) productive talk moves for supporting classroom discussions. Just these pages alone could provide a reinvigoration of mathematical reasoning in your classrooms!

The authors and contributors offer practical advice for using formative assessment strategies including designing rubrics. This advice would be most useful for moderation of students' work samples and for planning learning experiences.

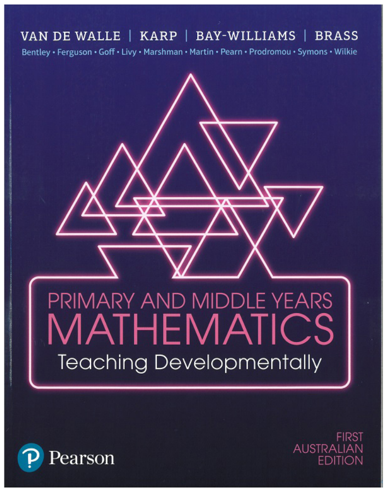
Although 'the internet is a wellspring of information' (Ven de Walle et al., 2019), it can also be a minefield (p. 142). Helpfully, this text provides references to digital tools and technology resources throughout the chapters, and also a list of recommended online resources, such as Illuminations (NCTM), youcubed, and Count On, in Chapter 7 on pages 142 and 143.

SECTION 2

The features of the chapters of the mathematical content section include learning objectives, 'big ideas', and many quick activities that are easy to read and administer. These activities are mapped to the Australian Curriculum. 'Guess My Rule' (Van de Walle et al., 2019) will be a familiar favourite (p. 538). Immediately evident due to the layout and organisation of the text is: notes about this activity, an indication of suitability for EAL/D students and students with disabilities, where it belongs in the development of data analysis skills, an illustrated example, similar activities, and links to the proficiency strands.

Many teachers have welcomed the clear diagrams of powerful pedagogical ideas in this text. One example are the graphics accompanying the thinking about how large numbers can be conceptualised, such as Figure 11.13 and Figure 11.14 copied below (Van de Walle et al., 2014, p. 248).

All chapters in the text provide relevant formative assessment notes, reflections, and resources. I am particularly drawn to the 'literature connections', where picture story books, such as *Counting on Frank* by Rod Clement, are mentioned and validated as relevant resources.



The appendices are very helpful and handy to have in hard copy - NCTM Mathematics teaching practices: from principles to actions, a description of the proficiencies, the AITSL Australian professional standards for teachers, a guide to the use of the blackline masters and the activities (mentioned previously) mapped to the Australian curriculum by year level and content description.

REFERENCE

Van de Walle, J., Karp, K., Bay-Williams, J., Brass, A., Bently, B., Ferguson, S., Goff, W., Livy, S., Marshman, M., Martin, D., Pearn, C., Prodromou, T., Symons, D. & Wilkie, K. (2019) *Primary and Middles Years Mathematics: Teaching developmentally. Pearson.*

This book is available from MAV's online shop, www.mav.vic.edu.au/shop.

INVESTIGATIONS

Ellen Corovic, Jennifer Bowden, Helen Haralambous and Danijela Draskovic – Education consultants, Mathematical Association of Victoria

FOUNDATION - YEAR 2

WEATHER WEAR

Adapted from FUSE, Department of Education

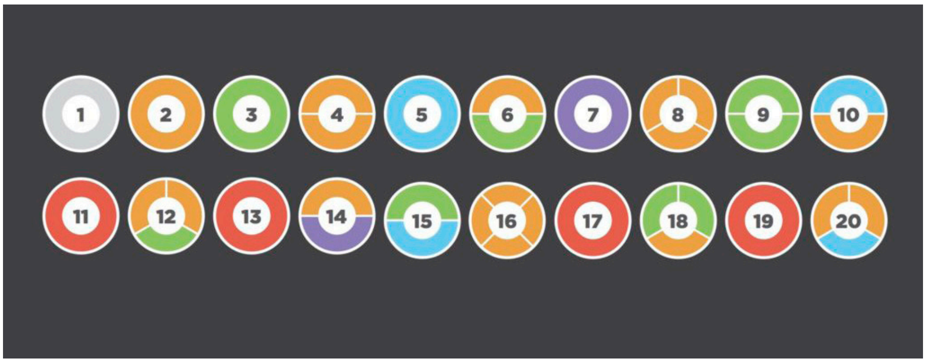
Watch the video https://education.abc.net.au/home#!/media/2035485/seasons-with-dirtgirl. What does the weather feel like during each season (summer, spring, autumn and winter)? What type of clothes do you need to wear? For each of the seasons create a picture of yourself in an appropriate outfit to suit the weather.

YEAR 3 - YEAR 6

PRIME CLIMB

Look at the number chart. Write down any patterns that relate to the colours in the chart. What pattern do you notice about the colours of the circles and way each circle is divided?

Create a rule for each colour. Use your rules to colour the number chart at https://mathforlove.com/lesson/prime-climb-color-chart/.



YEAR 7 - YEAR 9

ORDERING INDICES

Create cards like the ones here on the right. These are expressions written in index form.

Place each card in ascending order (smallest to largest).

Create your own set of nine cards containing expressions using indices, for a classmate to put in ascending order.

Enabling prompt: You may need to simplify the expressions.

$5^0 \times 5^2$	$\dfrac{10^4}{10^3}$	$\left(9^2\right)^2 \times 9^5$
$\dfrac{4^2 \times 4^5}{4^3 \times 4^4}$	$\dfrac{\left(3^8\right)^4}{\left(3^2\right)^8}$	$2^4 \times 2^2 \times 2^3$
$7^2 \times 7^9$	$3^2 \times 3^2$	$\dfrac{\left(6^2\right)^2 \times 6^{10}}{6^5 \times 6^7}$

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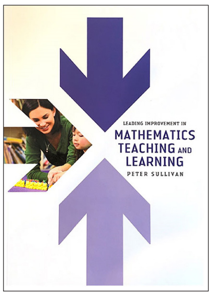


LEADING IMPROVEMENT IN MATHEMATICS TEACHING AND LEARNING

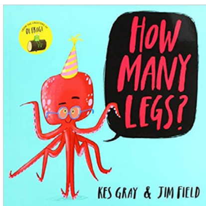
F - VCE

Schools have access to assessment data, motivating efforts to improve the numeracy outcomes of their students, but it can be difficult for principals to decide how to achieve these goals within their school's existing strategic plans and policies.

Emeritus Professor Peter Sullivan brings his decades of experience and research to a carefully curated selection of proven practices and effective approaches that will help school leaders empower teachers and achieve improved numeracy outcomes for their students.

Quality learning experiences, lesson structure, learning sequences, classroom culture, collaborative planning and effective teaching are addressed in ways that will help school leaders improve not only students' numeracy and mathematics outcomes, but also their critical and creative thinking skills, enabling classroom learning to be transferred to real-life contexts. This book provides a framework for a set of high-impact strategies that individually and together can be the focus of teacher professional learning, school improvement and student numeracy achievement.

$42.80 (MEMBER)
$53.50 (NON MEMBER)



HOW MANY LEGS

F-4

This book has got legs - the question is, just how many are there? In this zany, laugh-out-loud counting book, little readers are challenged to figure out how many legs there would be in a room if lots of many-legged friends showed up. How many legs would there be if a dog walked in with a chimpanzee? Or a frog hopped in on a kangaroo? Or a squid rode in on a buffalo? Hilarious illustrations and fun text ratchet up the giggles as each new friend joins the party. Can you keep track of all the wacky, multiple-legged 'guests' that turn up on the page? Maybe or maybe not, but you'll sure have a hoot along the way.

$14.60 (MEMBER)
$18.25 (NON MEMBER)

GAMES FOR GAMES DAYS

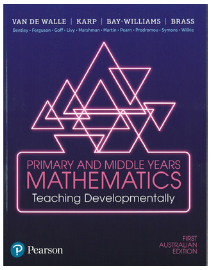
5 - 10

The Mathematical Association of Victoria (MAV) Games Days are very popular and a great way of engaging students through competing with like-minded individuals. MAV often gets enquiries from schools either wishing to run smaller scale games days at a local or school level or requesting games days resources.

MAV has compiled a selection of favourite maths games some used in games days. Whether for games days or for general classroom use, the games are a useful tool in engaging all students.

The resource has been designed with one game per page, so teachers can print the desired page as is. Each game has the same format, listing materials required, the rules and the aim of the game.

$20 (MEMBER)
$25 (NON MEMBER)



PRIMARY AND MIDDLE YEARS MATHEMATICS: TEACHING DEVELOPMENTALLY

F-9

This text is targeted towards teaching primary and middle years mathematics units in the Bachelor of Education degree.It illustrates how children learn mathematics, and then shows pre-service teachers the most effective methods of teaching mathematics through hands-on, problem-based activities.

The book serves as a go-to reference for the mathematics content suggested for Foundation to Year 9 as recommended in the Australian Curriculum: Mathematics (ACARA, 2016), and for the research-based strategies that illustrate how students best learn this content. This text presents a practical resource of robust, problem-based activities and tasks that can engage students in the use of significant mathematical concepts and skills. It reports on technology that makes teaching mathematics in a problem-based approach more visible, including access to ready-to-use activity pages and references to quality websites.

$124.95 (MEMBER)
$156.18 (NON MEMBER)

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