



## THE ART OF MATHEMATICS



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*Cam Plapp - Visual Arts Teacher, Nazareth Catholic Primary School, Grovedale and St. Mary MacKillop Catholic Primary School, Bannockburn*

#### DISCOVERING MATHEMATICAL WONDER THROUGH ART

I was a maths leader for ten years but in 2018, I decided to follow my passion for visual arts and become a full-time art teacher. I haven't left maths behind though – I've been delighted to weave the mathematics curriculum into my art teaching and have been able to demonstrate real-world connections and mathematical discovery to my students. Many of my students can see the beauty in art and are now exploring the elegance and beauty of mathematics.

During my 20 years as a maths leader and classroom teacher, I was regularly faced with concerns from other teachers on the pressures of covering such a broad mathematics curriculum, or connecting mathematics to real life concepts in a genuine and authentic way.

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# FROM THE PRESIDENT

Michael O'Connor

## THE COMMON DENOMINATOR

The MAV's magazine published for its members.

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Term 4 is conference time for the MAV. Staff and members of the conference committee spend a great many hours each year in preparation for this two day event in December. The theme for this year's conference is *Making Connections*. I believe that this is an apt description of conferences in general. They provide us with concentrated opportunities to connect with theory, connect that theory to practice and to connect ourselves to a wider group of our professional peers. For many of us it is also an opportunity to reconnect with friends and past colleagues.

Teaching is the profession of learning. We work to nurture learning in others but also continue to learn ourselves in and through our work. The broad aim of professional learning is to improve our capacity as teachers whether we are recent graduates, proficient, highly accomplished or leading teachers. Evidence of this improved capacity is seen in improvements to the learning outcomes for our students.

In recent decades much has been written about the nature of teacher professional learning. Guskey, Clarke and Hollingsworth, Desimone and Timperley to mention just a few, have each proposed models for thinking about teacher growth and change. One essential aspect in all of this work is the role of collaboration.

At this year's conference, as an aid and conduit for increasing collaboration

between members, we are introducing a new session: conversations with the president. Each year, one idea will be tabled by the president based on the level of discussion about it in the profession over the previous twelve months. A panel of key stakeholders will be invited to discuss the issue and formulate plans of action.

The topic for the panel discussion this year will be out of area teachers in secondary mathematics. The discussion will centre on establishing the size of this group and how to provide support for them in ways that are effective and meaningful.

Another important voice in these discussions is the MAV membership. There will be opportunity for questions from the floor at the time of the conference but not all members will be able to attend the conference. This is why I am inviting comment and questions prior to December, email president@mav.edu.au.

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# THOUGHT LEADERSHIP

MAV recently released *Valuing Mathematics in Society: a discussion paper*. MAV works within an evolving context, and at a crucial time in education, and in representing the needs of mathematics educators MAV has produced this paper to outline areas for discussion and action within four broad categories:

- Society and government
- School leaders
- Teachers as individuals and team members
- Students and families.

This paper elaborates on the selected areas as a discussion starter, identifying areas where we can advance mathematics education in Victoria for the decades ahead. Further papers that take a deep dive into specific areas will follow. This allows stakeholders to engage in considered debate and action where required. MAV is seeking feedback on the discussion paper, please read it and take the short five minute survey using the link in the paper.

Access the discussion paper at: [www.mav.vic.edu.au/Services-and-News/Advocacy](http://www.mav.vic.edu.au/Services-and-News/Advocacy)

# MAV PROFESSIONAL DEVELOPMENT

During Term 4 2019, a variety of presenters and MAV's own mathematics educational consultants will present workshops focussing on innovative teaching practice.

Make sure you reserve a place by booking online early, [www.mav.vic.edu.au/pd](http://www.mav.vic.edu.au/pd).

TOPIC	DATE	YEARS	PRESENTER
Mental computation: a priority in Victorian primary schools?	16/10/19	F - 6	Peter Sanders
Maths300: Taking it further	22/10/19	F - 6	Marissa Cashmore
Investigative approaches	23/10/19	F - 6	Martin Holt
Warm ups!	29/10/19	7 - 10	Helen Haralambous
Challenging students to think	7/11/19	7 - 10	Danijela Draskovic and Helen Haralambous
Creating challenging problem solving tasks to engage all students	21/11/19	F - 6	Michael Minas

## ROAD ACCIDENT ANALYSIS

Andrew Stewart

### DESIGNING SACS FOR FURTHER MATHEMATICS

I've been involved in Further Mathematics since it started in 1994, and have often been asked where the inspiration comes for SAC activities/questions. The need for activities with an open-ended aspect in the latest Study Design has increased the difficulties of this process. Inspiration has come from my wide reading range – general news, sport, business and science – and from my interaction with colleagues, in particular my colleague in writing StartPoints for many years, and more recently Trial Exams for MAV, Fiona La Trobe. With my retirement in 2018, I have more time to ponder many ideas for SACS. My current inspiration is the use of questions from past MAV Trial Examination papers to form the underlying basis for a SAC question/activity. The VCAA Examination papers, or other commercially available Trial papers, could also be a fruitful source of inspiration.

### ROAD ACCIDENT ANALYSIS

In January 2019, as in January 2017, the major headlines in an otherwise quiet month concerned the apparent increase in the Victorian road accident fatality rate compared to previous years. In 2017, these headlines inspired the questions for the Data and Analysis topic in the MAV Trial Exam 2. The international data came from the World Health Organisation (WHO) 2015 report, Australian time series data

from Gapminder and the Victorian fatalities data came from the Victorian TAC website.

WHO has released its 2018 report, but much of the Gapminder data set has not been updated. The Victorian data from the TAC has to be collected and recorded year-by-year from its website.

#### 2017, Exam 2, Question 1

The dot plot shows the distribution of road fatality rates, in deaths per 100 000 people, for a sample of 23 countries.

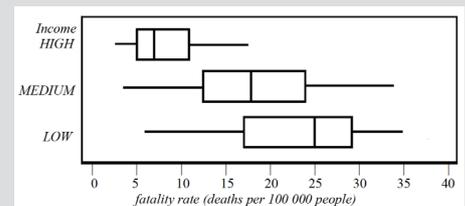
a) Circle, and clearly label, the point that is:  
i) Q1 ii) median iii) Q3

b) Use the grid to construct a histogram that displays the distribution of traffic fatality rates for this set of data. Use an interval width of one, with the first interval starting at 3.

#### 2017, Exam 2, Question 2

This table contains the traffic fatality rates (fatalities per 100 000 people) for 20 Asian countries, including Australia and New Zealand.

a) Show that the value of the upper fence is 29.5.  
b) Draw a boxplot of this data on the grid provided.  
c) In which part of the boxplot does Australia's value lie?



for each of these regions, or others of your own design that are deemed more suitable, could then be shared, and analysed, late in the SAC. Students could randomly select a specified number of countries from each of these regions, or from each of the three income groupings that have been allocated to each country to form their sample. Students could randomly select their own sample from within the entire data set, or use data sets set up for them.

The Gapminder dataset could be used to examine a particular country's fatality rate over a number of years up to 2007. This dataset has not been updated for quite a while, but does contain detailed information for a large number of countries.

Another alternative, which will require some internet research, would be to examine and compare the fatality rates in different parts of one country. For example, there are 50 states in the United States, ten provinces and three territories in Canada, and six states and two territories in Australia.

In presenting the material from the MAV Trial Exam 2 2017, I have left out many

# ROAD ACCIDENT ANALYSIS (CONT.)

graphs and tables to focus on the questions set and how they could assist in SAC design.

A good starting activity for the Data Analysis SAC is constructing a boxplot of the death rate values in the data sample, and then tabulating the key values, fence limits and standardised scores for outliers or highest/lowest values. Constructing a dotplot or a histogram (as given in Question 1, 2017) with specific column widths could assist in seeing the distribution or spread of the sample values, and assist in the design of the limits for, say, categories labelled for low, medium and high death rates. If these limits have already been set by the teachers, then students could discuss how their sample fits within these limits, or whether these are good limits to use. An alternative graphing activity could be constructing histograms to compare the natural and log values of the income values for each country in the sample.

## 2017, Exam 2, Question 3

The boxplots below (see page 18) display the distribution of road *fatality rates* (per 100 000 people) for countries classified as having *low* (less than \$6000), *medium* (between \$6000 and \$20 000) or *high* (above \$20 000) average income per person per year.

- Describe the shape of the distribution of fatality rates for the medium average income group.
- Using the information, explain the type of association that exists between fatality rates and average income. Quote the values of appropriate statistics in your response.

Describing the shape of a distribution displayed in a boxplot requires use of phrases 'approximately symmetrical' or 'positively/negatively skewed', and it's important for students to recognise which is appropriate for the graph displayed/drawn.

Explaining the type of association that exists between two variables using an appropriate statistic is one of the poorest completed parts of Exam 2 (see the VCAA Examiner Reports) and needs lots of practice. In this case, showing that a key measure has an inverse association with income changing from (using median values) 7 (Low) to 17 (Medium) to 25 (High) needs to be written clearly and succinctly.

Students could plot and analyse their own data sample in this way, providing that there are a reasonable number of countries in

each categorical group. As an alternative, this association could be analysed using a two-way frequency table where either students, or the staff preparing the SAC, convert the numerical death rate data into categories within defined limits.

A third alternative is a scatterplot featuring both the income data and the death rate data as numerical variables (both available in the WHO Report). This may also be a viable alternative to the time series approach shown below for aspects of bivariate analysis. It is more time efficient for the teachers to produce the scatterplot based on student data than for the students to plot the data for themselves, particularly if it is a large data set.

The bivariate data analysis questions in the Trial paper were based on time series data.

## 2017, Exam 2, Question 4

The traffic fatality rates (given as deaths per 100 000 people) for a twenty-year period for the state of Victoria are given in the table below, and a scatterplot of the data is shown alongside. The data will be used to investigate the association between the variables *fatality rate* and *year*.

- Use the scatterplot to describe the association between *year* and *fatality rate* in terms of direction, strength and form.
- Determine the equation of the least squares regression line that can be used to predict the *fatality rate* from the *year number*, where 1997 is *year number* 1. Round your answers to three significant figures.
- Draw the regression line on the scatterplot above. **Show** the calculations for the key points to place the line.
- Interpret the slope of the least squares line in terms of the variables *fatality rate* and *year number*.
- Assuming that the least squares regression equation from (b) can apply to data for 1994, predict the *fatality rate* for 1994, correct to three significant figures.
- The actual *fatality rate* for 1994 was 8.45. Calculate the residual value for your prediction, correct to three significant figures.
- What does this residual value tell us about the predicted value for 1994?
- Why is the prediction for 1994 unreliable?

This data was assembled by dividing the number of deaths each year as obtained from the TAC website by the estimated population for that year, and then converting the value to deaths per

100 000 people. For any other location – country or state – this calculation will be required from the number of deaths per year and the population for each year. Patient research on the internet will be required to find all the required information.

As mentioned, it is more time efficient for the teacher to produce the scatterplot (spreadsheets are very useful) based on the student data. Where the student is supplied with the data sample, the graph can be prepared ahead of time. Where the students have selected their own samples, some teacher homework time will be required to have the graphs ready.

Part (c), the drawing of the regression line, is an activity not well done by students. To improve their accuracy of line placement, ask students to show how they found the key points that place the line. These two points should be widely separated (at the edges of the graph?) and not close together in the middle.

While parts (c) and (d) may appear straight from an exam situation, they are important skills/abilities that students must display. Another important key skill/ability is the interpretation of the vertical axis intercept in a situation where it makes sense (not always possible). A good follow-up activity to all of these would require students to comment on the goodness of fit of the regression line to the data, and provide supporting diagrams where necessary. This, of course, brings in the 'three r's' –  $r$  (Pearson's correlation coefficient),  $r^2$  (coefficient of determination) and residuals (requiring a diagram). This type of question is generally not done well, and can provide an excellent separator for assessment of student ability and understanding.

Calculation of residual values is another area of difficulty for many students. Students could be provided with several values to complete their calculations. An alternative is to have students number their data sample from 1 to whatever, and then select a number of values by a carefully detailed random number-based process. They then calculate residuals for the values they have selected.

**2017, Exam 2, Question 5**

The time series plot shows the fatality rate per 100 000 people for the whole of Australia over a twelve-year period at about the time when stronger safety measures (such as seat belts) were starting to be implemented.

Five-median smoothing has been used to smooth the time series plot. The first four points are shown as  $X$ . Complete the five-median smoothing by marking smoothed values with  $X$  on the time series plot.

The data for both of these questions came from a Gapminder data set (Traffic deaths (per 100,000 people) which downloads with the title *RTI age adjusted indicator LIVE*), and used the values from within the range of 1960 – 1980 for Question 4, and 1980 – 2000 for Question 5. In the spreadsheet, there are sets of values for a number of other countries from 1950 to 2007 that could be used for similar tasks.

If the bivariate analysis involved ‘income’ as one of the variables, it is likely (but not certain) that a log transformation of the income value will be effective.

**2017, Exam 2, Question 6**

The traffic fatality rates for a twelve-year time period for the whole of Australia are given in the table below, and a scatterplot of the data is shown alongside. Note that the fatality rate value for the seventh year is NOT given.

For this data set, the relationship between fatality rate and time is non-linear.

A reciprocal transformation can be applied to the variable fatality rate to linearise the scatterplot.

a) Apply the reciprocal transformation to the data and determine the equation of the least squares regression line that allows the reciprocal of the fatality-rate to be predicted from the time. Round your answers to three decimal places.

b) Use this regression equation to predict the fatality rate in the seventh year. Write your answer to one decimal place.

It may be possible to create a number of data sets that require different transformations, and these can be distributed around the class.

Each student had their own unique data samples, rather than all students sharing the same sample.

To assess these, a spreadsheet was constructed which contained all the calculations and graphs and could access all the student sample values. Examples of these kinds of teacher spreadsheets can be found on MAV Further Maths StartPoints in 2013 and 2016.

Having a single data set that contains both numerical and categorical data (or the possibility of designing categories from the numerical data) makes it easy to start writing a SAC. Extra data of a specialised nature – annual fatality rates for countries or states, or annual population estimates – can be added to further enrich the SAC experience.

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# THE ART OF MATHEMATICS

Cam Plapp - Visual Arts Teacher, Nazareth Catholic Primary School, Grovedale and St. Mary MacKillop Catholic Primary School, Bannockburn

CONT. FROM PAGE 1.



Samples of Level 3-4 artworks that deal with shape (inspired by Klee).

Often, areas of the mathematics curriculum, particularly measurement and geometry, were 'brushed over', left to the end of term, or completely omitted altogether. I found this frustrating. As both as a mathematics leader and artist, I knew that rich and authentic real-world connections could easily be demonstrated to students.

## TAKING MATHS OUT OF THE MATHS CLASSROOM

I firmly believe that taking maths out of the mathematics classroom is a powerful way to make many areas of mathematics meaningful and authentic. This is particularly true when dealing with shape, symmetry, fractions and measurement. By taking these concepts out of the mathematics classroom and teaching them in the art room, I have found that we dig much deeper into the mathematics curriculum. There are strong (and sometimes very obvious) relationships between mathematics and the arts.

## Measurement, geometry and shape

**LEVEL 2:** Describe and draw two-dimensional shapes with and without digital technologies, provides an ideal opportunity to introduce students to artists such as Paul Klee or Piet Mondrian and allow students to develop and describe artworks using simple geometric shapes. Paul Klee's *Sun and Castle* (1928), Piet Mondrian's *Broadway Boogie Woogie* (1942) or Kandinsky's *Squares with Concentric Circles* (1913) are wonderful starting points for describing two-dimensional shapes and provide fantastic opportunities for students to create artworks inspired by famous artists (who also were inspired by geometric shapes!).

**LEVEL 3 and 4:** There are wonderful opportunities to explore symmetry in the environment and for students to create symmetrical patterns in art. Art lessons, such as creating mosaic tiles, drawing mirror reflections of simple objects such as leaves or feathers, or developing patterns (such as printmaking, which often explores concepts such as radial symmetry), provide ideal opportunities to make real-world

connections. Symmetry in art also provides fascinating explorations into other cultures such as Japanese Notan art, Greek mosaic and of course, Leonardo Da Vinci's *Last Supper* (1495-1498). One of my favourite art lessons has been creating styrofoam, lino or woodblock prints that explicitly explore transformations such as slides and turns. Using square tiles, I've explored concepts such as measurement and fractions, and the use of language such as half-turns/quarter-turns, also plays an integral part in the design process, and can help develop deeper connections between mathematics and the arts.

**LEVEL 5:** Students are required to 'apply the enlargement transformation to familiar two dimensional shapes and explore the properties of the resulting image compared with the original.' A very wordy description, which is summarised more directly in the Victorian Curriculum elaborations as, '... using a grid system to enlarge a favourite image or cartoon'. Outside my life as a teacher, I'm a professional mural artist and I cannot emphasise the importance of this skill! For any student wishing to pursue the



Samples of Level 3-4 artworks that deal with symmetry.



arts as they move into secondary education, I would urge them to explore and master the skill of grid systems.

**LEVEL 6:** Students are encouraged to investigate the effect of combinations of transformations on simple and composite shapes, including creating tessellations, with and without the use of digital technologies, another this is an ideal opportunity to explore the printmaking process. At this point, it would almost be an injustice not to introduce students to the genius of Dutch artist and mathematician, M. C. Escher when teaching this content of the mathematics curriculum!

### DEEPENING CONNECTIONS

The benefits of taking mathematical concepts into the art classroom are enormous. It gives opportunities to make deeper real-life connections between mathematics and the arts. It frees up time in the maths classroom by teaching concepts outside a normal mathematics curriculum. Covering the expanse of the crowded mathematics curriculum is not about working harder, but working smarter, and making genuine connections to the arts is certainly a way forward. For demonstrations of some of the art lessons mentioned in this article, subscribe to my YouTube channel, @camplapp.



A blank wall at our school provided inspiration for me to get my students involved in a large scale artwork that the whole school community could be proud of. We selected a scene from *Where the Wild Things Are* and then used mathematics to tackle the execution. Using an illustration from the book, we spoke about scale and perspective. A grid was drawn on the wall which helped students to break down the project into manageable chunks. The students loved this project as you can see by the happy faces on the cover of this magazine.

# USING THINKING ROUTINES

Sarah Ryan - Leading Teacher, Clifton Hill Primary School

A couple of years ago, I attended a Cultures of Thinking PD at Bialik College run by Ron Ritchhart from Harvard Graduate School of Education's Project Zero. 'Cultures of Thinking' are described as 'places where a group's collective as well as individual thinking is valued, visible, and actively promoted.' It was the most valuable professional learning I had experienced in a long time and the whole concept really resonated with me as a way to improve my teaching practice. Creating a culture of thinking is about getting more students doing more thinking more of the time. It's about developing rich understandings and creating opportunities for students to show their thinking in a variety of creative ways. Developing a culture of thinking involves fostering the eight cultural forces that shape group culture. The cultural forces are:

- Time
- Opportunities
- Routines and structures
- Language
- Modeling
- Interactions and relationships
- Physical environment
- Expectations

## WHAT ARE THINKING ROUTINES?

Thinking routines are simple structures, for example a set of questions or a short steps, that can be used across various grade levels and content. What makes them routines, versus merely strategies, is that they get used over and over again in the classroom so that they become part of the fabric of classroom culture. The routines become the ways in which students go about the process of learning.

## WHY USE THINKING ROUTINES IN MATHS?

Project Zero was initially focused on the arts and humanities. As I began to integrate Cultures of Thinking into my practice, I also started with English and Inquiry, as there seemed to be so many opportunities to use the different routines. As I became more familiar with thinking routines and saw my students become increasingly adept at using them, I began to consider how else I could use them and began to experiment in my maths lessons.

See	Think	Wonder
<ul style="list-style-type: none"> <li>• In between 0 and 1 there are more numbers called fractions</li> <li>• I can see quarters</li> <li>• The spaces in between the quarters are all equal</li> <li>• Half is exactly in the middle of zero and one</li> <li>• One quarter is halfway between zero and half</li> <li>• Three quarters is halfway between half and one</li> <li>• One half and two quarters are on the same line, they are the same</li> </ul>	<ul style="list-style-type: none"> <li>• It makes me think there are more numbers in between numbers like 0 and 1</li> <li>• One half and two quarters are on the same line so it makes me think there are different names for the same fraction</li> <li>• It makes me think of the football because there are 4 quarters and half time</li> <li>• It makes me think of a pizza when we cut it in half, then we cut the halves in half to make quarters.</li> </ul>	<ul style="list-style-type: none"> <li>• I wonder why we need fractions</li> <li>• I wonder if there are more numbers in between 0 and 1</li> <li>• It makes me wonder if there are other ways to say one quarter, one half and three quarters</li> <li>• I wonder if I could make the number line longer</li> <li>• I wonder if there are more numbers in between 0 and one quarter.</li> </ul>

Using thinking routines in maths is a great way to shift focus away from rules and procedures, placing importance of visualising and explaining thinking, and valuing the process, not just the 'answer'. I have found using thinking routines in the maths classroom has helped me to move away from a situation whereby as the teacher I ask a question, hands are raised and one child gets to answer. They are a great way to help students to question and understand why, to be critical thinkers and to make connections. Thinking routines compliment explicit teaching of mathematical concepts, and in my experience help to engage students who may feel they are not good at maths.

Using thinking routines allows authentic collaboration between students. Rather than students believing that others have copied their idea, building upon the ideas of others is encouraged. Students become more competent with these routines the more they are exposed to them in a variety of contexts. Using the same routine in reading, maths, inquiry and art for example, helps students to see that critical thinking skills are transferable even when the content is different. Thinking routines are best not viewed as activities, but as systems, procedures and techniques we can use to guide our thinking.

Many thinking routines work well in the maths classroom, and once you start to

experiment, the possibilities are endless. I have found that when introducing a new routine, doing it altogether as a whole class activity with the teacher recording responses on the board is helpful as it gives students the chance to hear how others respond and to have the routine modelled to them. The routines can all be done as a whole class, in small groups or individually.

The following are some of the many thinking routines that I have found to work well in the maths classroom:

## SEE-THINK-WONDER

In this routine, students are asked what they see, think and wonder about something. I would recommend giving students time to complete the *see* section before moving onto *think* and then *wonder*. The *see* section can surprisingly be the hardest as it's tempting to start saying what you think about something, rather than simply making observations. The *see* section encourages students to slow down and notice things they may not initially see. I find that using *See Think Wonder* as a whole class encourages students to build on others' ideas and delve deeper.

### Suggestions for use:

- Looking at timetables – e.g. a train timetable or TV guide
- Looking at a vertical algorithm
- Fractions on a number line

- Locating particular points on a grid reference
- Looking at a data set or data represented as a graph

### TEXTA TALK

This routine involves the teacher writing open questions on butcher's paper. I usually do several different questions, one per sheet. The students then walk around the room, silently writing their responses to the questions on each sheet. At this point I encourage students to remain quiet and let their texta do the talking. I find it helpful to have a group at each sheet, then ring a bell after two minutes for the groups to rotate to the next question. When each group has visited each sheet, students can be given a few minutes to walk around and look at all the recorded ideas. In this routine, adding to the ideas written by others is encouraged. Texta talk can be a great group pre-unit assessment allowing the teacher to gauge what students already know and even identify misconceptions. It's also a great way for students to recall what they already know about a topic and for the ideas of others to help them to remember things they may have forgotten.

#### Suggestions for use:

- Introducing time (questions could include why do we need to tell the time?, what are some activities that take about an hour?, what do you see on an analogue clock?, how do analogue clocks work? etc.)
- Introducing length (questions could include what are some units we could use to measure length?, what are some items that measure around 1 metre?)

### THINK-PUZZLE-EXPLORE

This routine encourages students to link prior knowledge to new learning by recording what they currently think, what puzzles or questions they have and how they may explore the concept further. The puzzle and explore sections are a valuable way to generate questions about a new topic to be addressed through the unit.

#### Suggestions for use:

- What is multiplication?

- What is a fraction?
- Are fractions related to division?
- What is perimeter?
- Analogue time

### CONNECT-EXTEND-CHALLENGE

This routine is somewhat similar to *Think Puzzle Explore*, but I would tend to use this one after introducing a new topic, giving students the chance to make connections to prior knowledge and also reflect on how their thinking is changing. In this routine students are asked to connect a new idea or concept with prior understandings, identify what aspects of this new idea have extended them and recognise what has been a challenge so far. This routine works well initially when introducing a topic that has clear links to prior learning, for example dividing fractions when students already know how to divide whole numbers. With practice, students will become more competent at making less obvious connections.

#### Suggestions for use:

- Adding and subtracting decimal numbers
- Dividing fractions
- Calculating the area of a triangle
- 24 hour time

### CLAIM-SUPPORT-QUESTION

In this routine, students take a claim (that may or may not be true), provide support for that claim and identify what questions are raised by that claim. Students can be provided with a claim by the teacher, or can think of their own.

#### Suggestions for use:

- Propose an idea that captures a misconception, e.g. all numbers end with a zero when multiplied by 10, a third is the same as 0.3, all fractions are smaller than 1 or when a number is multiplied by another, when a number is multiplied, we get a bigger number
- Alternatively the claim may be questionable or true, for example if I put nine red balls and one black ball in a bag and pull one out, I will probably pick out a red one, or *Snakes and Ladders* is a game of chance.

### CONCLUSION

Using thinking routines in the maths classroom can provide a refreshingly different approach for students and teachers. These routines and more can be used when introducing a new topic, reflecting on the outcome of a game, as an assessment tool, to generate questions and to share ideas. The benefits of using thinking routines in maths include collaboration, engagement, creative and critical thinking and making observations and connections. I find the structure they give to the thinking process really helps my students to delve deeper, both individually and as a group.

There is a lot more to Cultures of Thinking than thinking routines alone and I would recommend attending a PD session and reading to find out more.

### FURTHER READING

Thinking routines resource from Project Zero: [www.visiblethinkingpz.org/VisibleThinking\\_html\\_files/03\\_ThinkingRoutines/03a\\_ThinkingRoutines.html](http://www.visiblethinkingpz.org/VisibleThinking_html_files/03_ThinkingRoutines/03a_ThinkingRoutines.html)

Ritchhart, R., Church, M., Morrison, K., *Making Thinking Visible*, John Wiley & Sons Inc (US), (2011)

Ritchhart, R., *Creating Cultures of Thinking*, John Wiley & Sons Inc (US), (2018)

[www.pz.harvard.edu/projects/cultures-of-thinking](http://www.pz.harvard.edu/projects/cultures-of-thinking)

[www.visiblethinkingpz.org/VisibleThinking\\_html\\_files/03\\_ThinkingRoutines/03b\\_Introduction.html](http://www.visiblethinkingpz.org/VisibleThinking_html_files/03_ThinkingRoutines/03b_Introduction.html)

# CLASSPAD AND FURTHER MATHS

Kevin McMenamin - Head of Mathematics, Mentone Grammar School

## ROUTINES AND STANDARD APPLICATIONS WITH CASIO CLASSPAD

Digital technology use in both of the Further Mathematics examinations is very beneficial and has the potential to save valuable time. Getting to know the functions and applications requires commitment and practice in order for their use to become obvious and repetitive.

Beginning with the Core study of Data Analysis, many of the routine and standard applications linked to finding summary statistics or displaying the data in a graphical form are evident via the Statistics App. Examination questions generally require linear regression equations to be found, transformations to be applied and residual investigations to be undertaken. A question from the 2018 examination required a reciprocal transformation to be applied.

After entering the data into columns, the transformation is added via the row labelled **Cal**, located at the base of the columns (Figure 1).

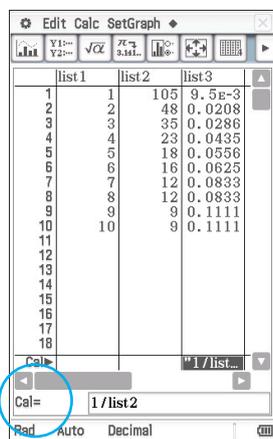


Figure 1.

When entering the reciprocal transformation into this cell, the coding requires reference to the column name, which is included automatically in the default set-up of the App. This can either be typed from the keyboard or copied from the column header to allow the transformed values to be created. The new linear regression can then be found using the drop down menu system (Figure 2) and referencing 'list 1' and 'list 3' as the required columns of data.

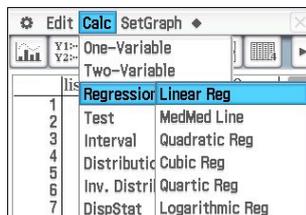
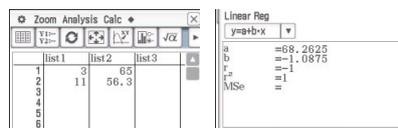


Figure 2.

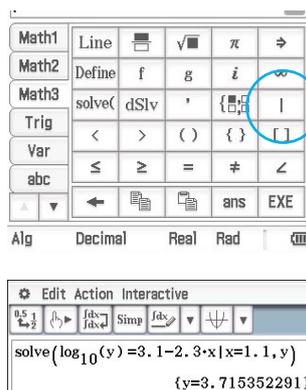
Consistent application of processes should be the goal of all uses. This would ensure repetition and assist the user in retaining the information in their long term memory.

Some particularly useful examples are linked to finding equations and calculating values of straight lines. An examination question from 2018 required the calculation of a linear regression equation from a sketch. After locating the coordinates of two points, they are entered into the **Statistics** App (Figure 3a) where a linear regression equation can be quickly and efficiently found (Figure 3b).



Figures 3a and 3b.

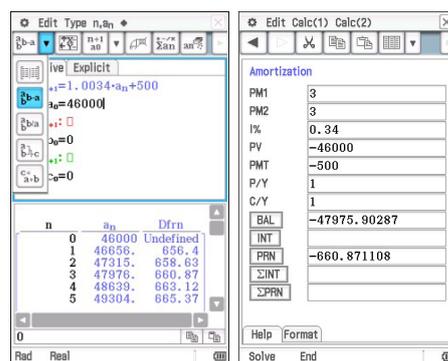
Another 2018 Examination question required the calculation of the response variable of a transformed regression equation. By using the 'with' operator, available on the Math3 palette (Figure 3c), substitutions and solving processes are made easy. As the regression equation was transformed, it is recommended the entire transformed equation be entered into the **Main** App first, followed by the 'with' operator and then the appropriate substitution. The calculator when then solve the equation as required (Figure 3d).



Figures 3c and 3d.

## RECURSION AND FINANCIAL MODELLING (SEQUENCE APP)

When the new 2016 study design for Further Mathematics introduced Recursion and Financial Modelling as a core component, the **Sequence** App available on the ClassPad provided a set of functions and outputs that linked directly to the content involved and made the answering of many questions a little easier. One example from the 2018 examination wanted to know the difference between two terms of a sequence which was described using a recursive equation. Via the drop down menu linked to the ordered pair button (left hand side of the toolbar), there are a series of calculating options to choose, including a difference between successive terms. When activated, this list is displayed as a new column alongside the ordered pairs (Figure 4a).



Figures 4a and 4b.

Another option to find this same solution is through the use of the Amortization table (Figure 4b), which is part of the **Financial** App. The recursive equation information can be converted into values appropriate for the table and the principal increase then found. This option is particularly useful for questions investigating the total of payments made or interest paid over a period of time.

The **Sequence** App also finds explicit rules from recursive equations. A question from the 2016 Examination required an explicit equation to be found from the provided recursive equation. Selecting the *sequence run* button from the toolbar (see right) and using the *rsolve* function (Figure 5) enabled the conversion to be made.

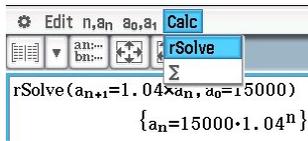
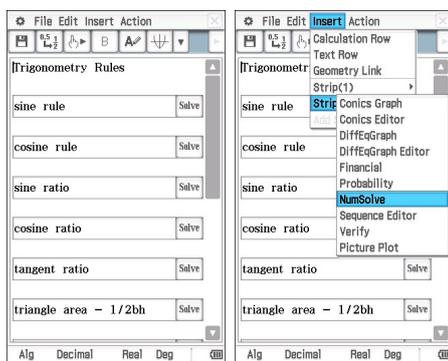


Figure 5.

The other recursive notations ( $a_{n+1}$ ,  $a_n$ ,  $a_0$ ) needed to construct the equation is readily available from the menu items at the top of the screen.

### USING AN E-ACTIVITY PAGE TO STORE RULES

There are many rules that are used repetitively and would benefit from being stored in a location that is readily accessible and cannot easily be deleted. Geometry and Measurement, Data Analysis and Linear Relations contain many rules that would suit this App (Figure 6a). A majority of the rules would be entered via the **Numsolve** application which is available through the Insert menu (Figure 6b).



Figures 6a and 6b.

Each new window allows a rule to be entered using parameters (Figure 7). These parameters immediately become available in the lower half of the window where values can be assigned, leaving one to ultimately be solved.

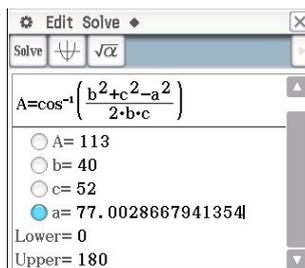


Figure 7.

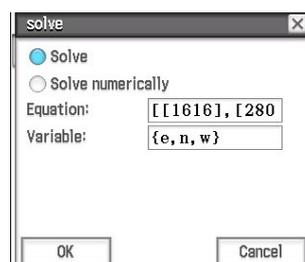
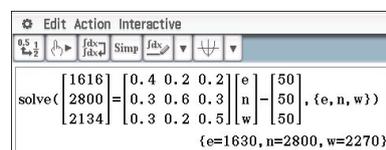
Selecting the circle adjacent to the parameter to be solved and then selecting the Solve button from the toolbar will complete the process. Pages like this can then be saved and used when appropriate, similar to saving files on a computer.

### LATEST OPERATING SYSTEM

The Casio Research and Development team is always looking for ways to improve the technology through new functionality options and improved processing capabilities. It is imperative that when a new operating system becomes available, it is immediately installed on the handheld device and emulator. The latest operating system, 2.01.6, was released in Term 2, 2019 and is available at Casio's Education website, <https://edu.casio.com/en>. New features have been added and algorithms have been adjusted to improve function calculations and evaluations.

One of the recent additions to the operating system is the capacity to solve matrix equations without the use of an inverse. A multiple choice question on the 2018 Examination 1 used a recurrence equation of the form  $A_{n+1} = TA_n - D$  and required the previous state matrix to be calculated.

The formatting uses standard matrix templates (Figure 8a) available from the Math2 palette of the virtual keyboard. When the solving process is activated through the **Interactive** menu, the variables are entered nested in a set of 'curly brackets' separated by a comma. (Figure 8b).



Figures 8a and 8b.

Features like this are part of an extensive series of 'how to' videos available on the Casio Education website, [www.casio.edu.shriro.com.au/app/classpad\\_how\\_to\\_videos.php?section=all&module=95](http://www.casio.edu.shriro.com.au/app/classpad_how_to_videos.php?section=all&module=95)

These videos provide an excellent resource as they work each function into an example and describe, in detail, how they are applied thus making it a wonderful resource for users to improve their knowledge of their device.

# TI-NSPIRE AND FURTHER MATHS

Peter Flynn - Texas Instruments

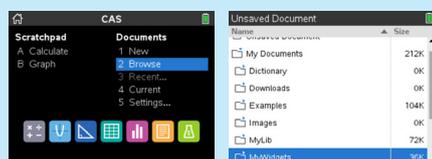
## USING TI-NSPIRE CX CAS WIDGETS AND NOTES TO OPTIMISE EXAM RESULTS AND ENHANCE LEARNING

As illustrated in the last issue of *Common Denominator*, the TI-Nspire CX CAS Notes application is an interactive environment that allows mathematical calculations to be performed with accuracy and efficiency and provide rich, collaborative and dynamic mathematical experiences for learners. This interactivity stems mostly from the facility to embed linked mathematical expression boxes (known as Math Boxes) within a text document. These Math Boxes automate numeric and symbolic calculations and are very effective in answering exam questions.

In this article, this little-known interactivity will be demonstrated through an exam-style example involving transition matrices and in a brief classroom study of a remarkable data set known as Anscombe's quartet.

## INTRODUCING TI-WIDGETS

For answering exam questions requiring accurate and efficient calculations, a TI-Nspire CX CAS offers the powerful facility to create, save and use a one-page document known as a Widget. With TI-Nspire CX CAS, all work created and saved with TI-Nspire CX CAS applications are stored as a document which can be shared with others. A Widget is a TI-Nspire (.tns) document that is stored and accessed in a user's MyWidgets folder. This folder can be located as shown in these screenshots.

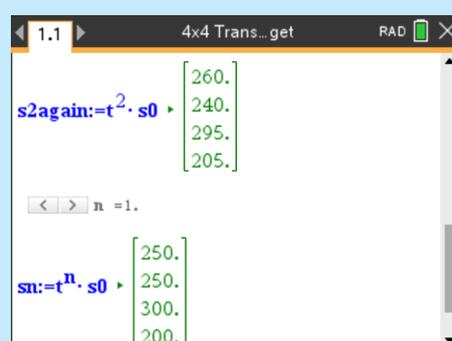
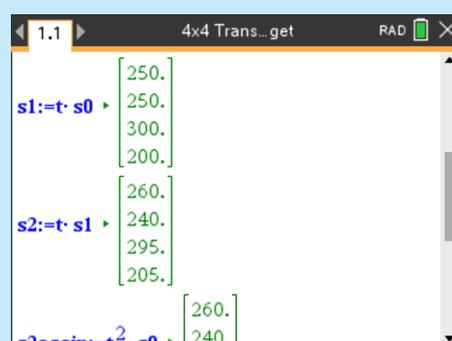
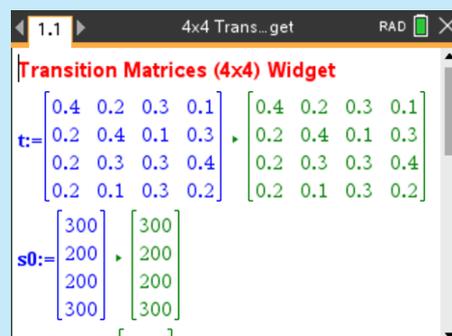


A document is only regarded as a Widget when it is saved or copied to the designated MyWidgets folder. Widgets can be used to seamlessly access text files, insert and run pre-prepared templates that automatically answer suitable exam questions and insert a saved problem into a document. When a Widget is added, TI-Nspire CX CAS extracts only the first page of the selected TI-Nspire (.tns) file and inserts it into the open document. Hence it is advantageous for a Widget to form a lean design and consist of a single page only.

Prior to sitting a Further Mathematics exam, teachers and students have the opportunity to devise interactive Widgets with embedded Math Boxes to readily answer heavily-templated exam questions. Examples of numeric and symbolic calculations suitable for a Widget include statistical, matrix and trigonometric calculations.

## CREATING, SAVING AND ADDING A TI-WIDGET

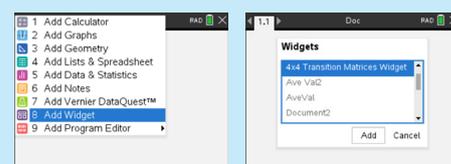
Let's suppose you wish to create a Widget for an upcoming CAS-permitted exam that makes use of the matrix recurrence relation  $S_{n+1} = TS_n$  where  $T$  is a transition matrix ( $4 \times 4$  in this case) and  $S_n$  is a column state matrix. The initial state matrix is denoted by  $S_0$ . After adding a Notes application, set up the Widget as shown in the following screenshots. The Widget shows the matrices  $T, S_0, S_1, S_2$  and  $S_n$ .



To insert a Math Box, press **Menu > Insert > Math Box**. Note the use of the assign command ( $:=$ ). The input is displayed in blue and the output is displayed in green. **Menu > Math Box Options > Math Box Attributes** allow the user to change the settings and appearance of a Math Box. Surrounding explanatory and prompting text can also be added to a Widget to provide enhanced understanding of what the Widget is doing or offer advice on how it should be used. For example, to obtain a floating point decimal answer form rather than an exact answer form. To insert a slider for  $n$  that allows the user to control the value of  $n$  and hence the number of transitions, press **Menu > Insert > Slider** and complete each field as desired.

To save the Widget to the MyWidgets folder, navigate to **My Documents (Browse in version 5.0) > MyWidgets**, type in a name for the Widget and save. If an exam question asks candidates to consider a  $4 \times 4$  transition matrix, access the Widget in one of the two following ways.

To add a Widget to a New Document, open a **New Document**, click **Add Widget**, scroll to select a .tns file from the box and click **Add**.



To add a Widget to an existing document, press **Doc > Insert > Widget**. Now all the student has to do is correctly interpret the exam question and fill out the Math Boxes requiring alteration (remembering to press **Enter** each time). If the exam question is worth more than one mark, students must show appropriate working. Students must also be cognisant of answer form if one is specified.

## ANSCOMBE'S QUARTET: PEDAGOGICAL USE IN THE CLASSROOM

Anscombe's quartet comprises four bivariate data sets (11  $(x, y)$  points) with nearly identical statistical properties, yet appear very different when graphed. English statistician, Francis Anscombe (1973), constructed these data sets to show

the importance of graphing data prior to analysing it and to show how outliers can affect a data set's statistical properties. Anscombe's four data sets are displayed in the table below. The 4 data sets are  $(x_1, y_1)$ ,  $(x_1, y_2)$ ,  $(x_1, y_3)$  and  $(x_2, y_4)$ .

$x_1$	$x_2$	$y_1$	$y_2$	$y_3$	$y_4$
10	8	8.04	9.14	7.46	6.58
8	8	6.95	8.14	6.77	5.76
13	8	7.58	8.74	12.74	7.71
9	8	8.81	8.77	7.11	8.84
11	8	8.33	9.26	7.81	8.47
14	8	9.96	8.1	8.84	7.04
6	8	7.24	6.13	6.08	5.25
4	19	4.26	3.1	5.39	12.5
12	8	10.84	9.13	8.15	5.56
7	8	4.82	7.26	6.42	7.91
5	8	5.68	4.74	5.73	6.89

## EXPLORING ANSCOMBE'S QUARTET: CENTRAL TENDENCY AND SPREAD

The screenshots below, taken from a .tns file that can be readily sent to students with TI-Nspire CAS Navigator software, illustrate how Anscombe's quartet can be used to encourage meaningful discussion of a data set's graphical properties and associated statistical measures. Pages 1.4-1.6 of the .tns file are Notes application pages housing math boxes that display each data set's near identical statistical measures.

Use each **Math Box** below to calculate the **mean** and the **variance** of  $x_1$  and  $x_2$ .

$\text{mean}(x_1) = 9$   
 $\text{mean}(x_2) = 9$   
 $\text{varSamp}(x_1) = 11$   
 $\text{varSamp}(x_2) = 11$

What do you notice?

Use each **Math Box** below to calculate the **mean** of  $y_1, y_2, y_3$  and  $y_4$ .

$\text{mean}(y_1) = 7.50091$   
 $\text{mean}(y_2) = 7.50091$   
 $\text{mean}(y_3) = 7.5$   
 $\text{mean}(y_4) = 7.50091$

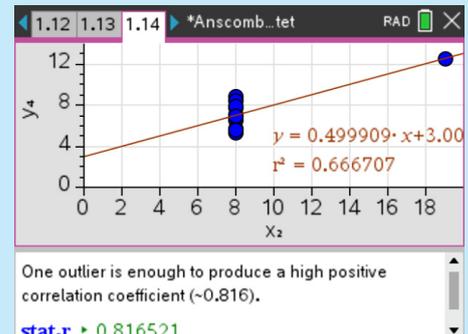
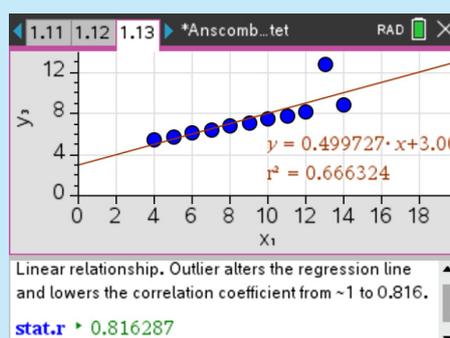
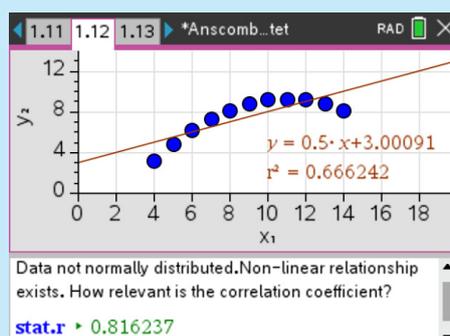
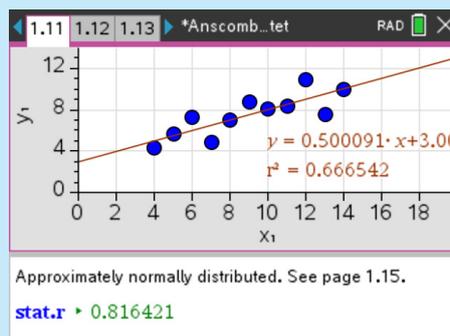
What do you notice?

Use each **Math Box** below to calculate the **variance** of  $y_1, y_2, y_3$  and  $y_4$ .

$\text{varSamp}(y_1) = 4.12727$   
 $\text{varSamp}(y_2) = 4.12763$   
 $\text{varSamp}(y_3) = 4.12262$   
 $\text{varSamp}(y_4) = 4.12325$

What do you notice?

Pages 1.11-1.14 show the four scatterplots with their linear regression equations. Each regression equation is  $y = 3.00 + 0.500x$  (correct to two and three decimal places respectively). Students also determine that each data set's Pearson's product-moment correlation coefficient is 0.816, correct to three decimal places.



At the start of this activity, ask students to sketch a scatterplot consisting of 11 data points and a Pearson's product-moment correlation coefficient of approximately 0.8. Then invite students to compare their scatterplots. The overwhelming majority will sketch a scatterplot similar to the one displayed on page 1.11. Subsequent analysis and discussion can ensue regarding what other scatterplots with similar statistical properties may appear like. These data sets are a terrific resource to discuss the effect of outliers e.g. pages 1.13-1.14 of the .tns file.

## CONCLUSION

Widgets allow students the opportunity to create 'live' revision notes for use in exams and hence answer exam questions accurately and efficiently. Interactive Notes pages consisting of explanatory text, diagrams, images and Math Boxes afford opportunities for teachers and students to create powerful learning experiences. With such functionality, it is a great time to be teaching, learning and doing mathematics.

## REFERENCES

Anscombe, F. J. (1973). Graphs in Statistical Analysis. *American Statistician* (27) 1: American Statistical Association.

# 2 YEAR COLLABORATIVE PROJECT

Amy Somers - Leading teacher of mathematics, Lyndale Greens Primary School

At Lyndale Greens Primary School we provide leadership opportunities to all our staff. We have six new level leaders who are very keen to learn and grow as professionals. When we heard about the Mathematical Association of Victoria's two year collaboration project we jumped at the chance to be involved so we could send our team of new leaders, our Principal, two learning specialists and myself, to develop all of our abilities to lead mathematics. Here's an overview of our journey so far.

## HOW WE STARTED

We attended two professional development days in February. We absorbed professional readings, reviewed our AIP and completed a number of activities to help us clarify our collaborative project goals. As a team we established why this focus would be beneficial to our educational setting.

## OUR GOAL

An ongoing school focus has been to develop teacher knowledge of the four proficiencies. We decided that we would need to work incrementally towards our goal and tweak current practice, rather than change too much of our already successful mathematics program. After a lot of discussion we decided to start by introducing *Talk Moves* at our school.

We focused on *Talk Moves* because:

- As a way to begin to focus on mathematical discourse and reasoning in our classrooms.
- So we could continue to develop greater consistency between our classrooms and develop a consistent mathematical language between students and staff.
- So teachers don't feel overwhelmed with too much being introduced at once.
- To eventually lead into introducing *Number Talks* and to use this as a vehicle to continue to develop teacher understanding of the four proficiencies.
- So that Specialist teachers could also be included, as *Talk Moves* can be implemented across subject areas.

Talk move	Explanation
Wait time	Giving students time to think about a problem is very important to help them develop their ideas. More wait time between the teacher asking a question and expecting an answer also allows children to think about the question on a deeper level.
Turn and talk	Students are often more comfortable sharing their ideas with a friend rather than the whole class. This talk move gives students time to rehearse what they want to say, clarify their thinking and sometimes even revise what they think based on what their partner has said.
Revoicing	'Are you saying...?' This talk move involves reiterating what a student has said and/or asking for clarification. Revoicing helps to clarify what a student has said which can highlight important ideas or reveal a misunderstanding. This is also a way a teacher could introduce mathematical vocabulary into the conversation or correct its usage.
Reasoning	This talk move encourages students to justify or elaborate on their thinking or someone else's thinking using evidence.
Adding on	Students are encouraged to add on to other students' ideas and agree or disagree with their ideas by justifying their own ideas.
Repeating	By getting students to repeat what someone else has said we get students to listen more carefully to each other. We can also use this talk move to add emphasis to important ideas and to slow the pace of a discussion.
Revise your thinking	After students have heard the explanations and ideas of their peers, they might discover something new and change their mind. This talk move emphasises the importance of changing your thinking once new knowledge is available or understood.

We took this to staff and asked everyone to have a go at implementing one *Talk Move* in their class by the end of the term.

## WHAT ARE TALK MOVES?

*Talk Moves* consist of a range of strategies that teachers can implement to facilitate class discussions. They help to teach students how to share, expand, clarify and justify their thinking. They also help students to be active listeners by getting them to tune in to what others are saying.

There are a lot of articles about *Talk Moves* and many of them use slightly different terminology. We decided to use the seven talk moves which Jennifer Way and Janette Bobis discussed in their article *The Literacy of Mathematics*. These are noted in the table above.

We used hand signals to help students communicate what they are thinking.

Students make a fist on their chest to show that they are still thinking. They then put their thumb up to show they have one way to work out an answer and then put up a

finger if they can find more than one way to solve the problem.

Our students are also using another gesture to indicate 'I agree' or 'me too'. The student makes a fist then sticks their thumb and little finger up and rocks their hand back and forwards toward the person they are agreeing with.



## OUR JOURNEY

We were a bit too keen with our February goal. Back at school there was also Literacy, Wellbeing, ICT and Data Literacy elements we were focussing on and on top of that we were undertaking our school review!

# IMPROVE MATHEMATICS OUTCOMES AT YOUR SCHOOL

Join the **Mathematics Collaborative 2020 – 2021** and lead whole school improvement in primary school mathematics education.

MAV and the Mathematics Education Group (MEG), at the Melbourne Graduate School of Education (MGSE), have developed a two-year school improvement program for primary schools in Victoria.

The program launched in 2019 with 39 primary schools from across Victoria from all sectors. Positive feedback from participating schools has been received, and due to the success of the program there is an opportunity to take in a second group of schools from 2020.

The program is also undergoing a formal evaluation by the team of researchers at The University of Melbourne. Over time, this will allow the program to further develop as one of the most rigorous and successful of its type.

## A TWO-YEAR SCHOOL IMPROVEMENT PLAN

The program allows schools to develop the tools and techniques to implement and measure improvement, while working with a network of like schools striving for improvement.

### Schools and school leaders in the Maths Collaborative will:

- develop a clear and practical focus on school improvement
- develop goals and implement change in a supportive environment
- engage with researched based methods and tools for improvement
- investigate and apply the tools and techniques to implement and measure improvement
- gain a practical understanding of the structures to support improved practice by working collaboratively within their school
- implement supportive and collaborative structures within their teams at school
- develop a deeper understanding of the maths proficiency strands and the teaching practices to support student development in mathematics.

### Teachers in schools undertaking the Maths Collaborative will:

- implement supportive and collaborative structures
- improve their mathematics content and content pedagogical knowledge while working collaboratively with colleagues
- have opportunities to network across schools, and engage in professional learning events
- strengthen their ability to cater for diverse ranges of student needs (differentiation)
- consolidate approaches to embedding the capabilities including critical and creative thinking
- work towards greater cognitive engagement.

Find out more at [www.mav.vic.edu.au](http://www.mav.vic.edu.au).

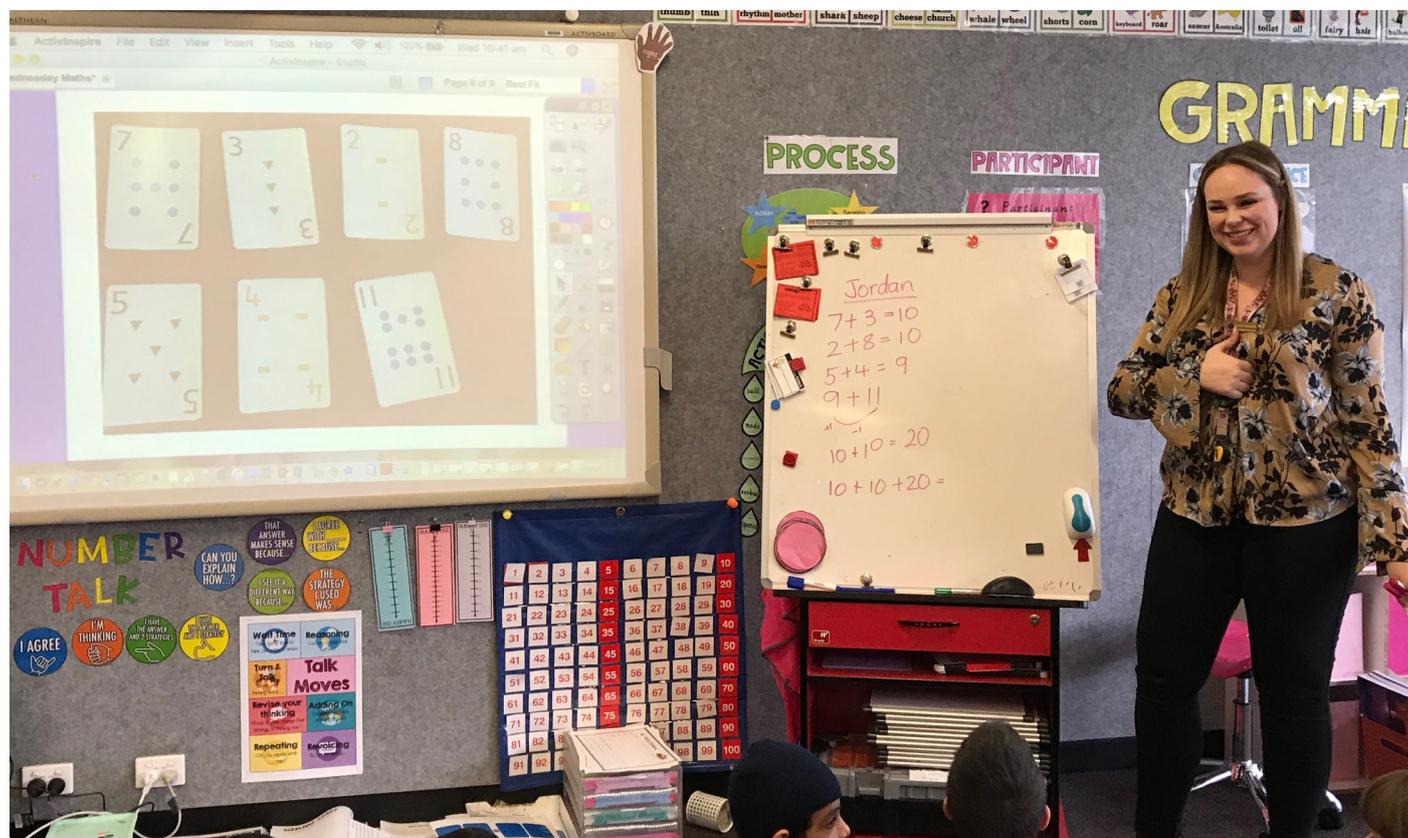


THE MATHEMATICAL  
ASSOCIATION OF VICTORIA



THE UNIVERSITY OF  
MELBOURNE

# 2 YEAR COLLABORATIVE PROJECT (CONT.)



For our next session in May, we all felt a bit like students who hadn't done our homework, since we hadn't achieved what we had initially set out to do. After some reflective discussions about what we had attempted in our own classes and within our coaching, and a review of the research/readings/resources that we had started to utilise, we realised we had actually done quite a bit. We were also able to share our journey with others undertaking their own collaborative projects and found that many schools were in a similar position.

At this session, our team refined our approach and decided on a manageable implementation of *Talk Moves*. We then set a group goal to trial one *Talk Move* in our class before introducing all of the moves to the rest of the staff.

Our next few sessions together were back at school, where we honed in on our focus by completing some professional readings supplied to us by our MAV consultant, Judy Gregg, and running PD for our level leaders. We also continued to monitor how we were tracking with our own implementation of *Talk Moves* and decided how we would run whole school PD to start

to get the rest of the staff involved. We discussed what we wanted to include as well as how we would establish reasonable expectations and timeframes for staff following the PD.

Our level leaders decided they wanted to show the rest of the staff what their early experimentations with *Talk Moves* looked like in our context rather than showing them a polished video that they had found online, so each level leader filmed themselves using an aspect of *Talk Moves* in their own classes.

We also asked our teaching staff to complete a self-evaluation on how they currently run mathematical discussions which we sourced from *Everything You Need for Mathematics Coaching*. We aim to use these evaluations as a form of evidence throughout our project. In the discussion that arose from these evaluations, many staff said that they'd like further professional development on how to improve their discussions and questioning in mathematics.

Judy Gregg visited again and further supported us with PD and helped to clarify the difference between *Talk Moves* and *Number Talks* as some of us had become

a bit confused with this. She also watched all our *Talk Moves* videos, provided us with some feedback and joined us on a tour of our school to see how we teach mathematics. After this Judy assisted in further refining our staff PD.

During our whole school PD session, our team explained our goals to the staff, introduced them to *Talk Moves* and shared their videos. At the end of the PD we asked all staff to choose a *Talk Move* to use for the next two months and to implement this in their teaching. All of our level leaders shared that while they had only aimed to introduce one *Talk Move* they naturally implemented several, because they were now conscious of them. It will be interesting to see if this occurs more widely amongst staff.

The staff PD went really well and we received a lot of positive feedback from our teachers. They really appreciated seeing videos of the *Talk Moves* in action in our school and in our context. Specialist teachers could see how they could use *Talk Moves* in their classes too and were keen to give them a go. Our teachers also appreciated that we were refining current practice rather than starting something

completely new. They felt that it was achievable to implement an aspect of *Talk Moves*, in line with the step-by-step timeline we had planned for the school.

It has been great to see our level leaders grow and develop their understandings of mathematics leadership and for the members of our SIT team to also grow and learn alongside them.

## WHERE TO NEXT

We continue to develop our knowledge and plan the next steps towards our overarching goal of increasing staff knowledge of the four proficiencies.

After our first six months of the Maths Collaborative Project we have half the staff actively involved in *Talk Moves* and showing signs of being ready for the next part of our plan to introduce *Number Talks*, with the other half wanting time to embed and deepen their knowledge of *Talk Moves* before moving on. I think this will work nicely with our group as we continue to grow together and support each other over the coming 18 months of the project.

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## TEACHERS INVOLVED IN THE MATHS COLLABORATIVE SHARE THEIR THOUGHTS

### What have you enjoyed?

The eagerness of aspiring young leaders to lead a high impact teaching strategy and up-skill staff through professional development has been a great highlight. I have enjoyed observing students build in confidence in an inclusive learning environment where students' thoughts are welcome and where they are expected to

speak, listen and respond to one another.

– Leonie Haggett, *Learning Specialist*

I have enjoyed hearing from experts in the field of mathematics who have an invested interest in the direction mathematics is taking. I have also enjoyed that it has been with an Australian context, as lots of readings and PDs often refer to examples or data that is outside of Australia.

– Tahlia Bowden, *Year 3 Level Leader*.

Being able to encourage each other to try new things and experiment with aspects of our teaching practice. I love that this project has raised the profile of maths.

– Kylie Smith, *Learning Specialist*.

### How have you grown as a maths leader?

I can now see clearly how through *Talk Moves* I am addressing many High Impact Teaching Strategies (HITS) and promoting problem solving and reasoning, which I knew I previously wasn't exposing or devoting enough time to.

– Leonie Haggett, *Learning Specialist*

Using *Talk Moves* has created more of a balance between teacher and student talk within my classroom. I have realised the importance of giving students time to think.

– Ashlin Sagers, *Year 5 Level Leader*

### What have you learnt?

It's important to clarify aims and purpose and map out an action plan to ensure what we are striving to achieve is manageable, implemented, measurable and reviewed.

– Leonie Haggett, *Learning Specialist*.

How to implement change across a school. It's been important to see the strategies to get buy-in from teachers and to implement small changes gradually.

– Olivia Penhalluriack, *Year 2 Level Leader*

### What difficulties have you come across how have you overcome these?

We need to take small steps in implementing something new and not overwhelm people. It's been important to keep it simple and achievable. – Daisy Frey, *Year 4 Level Leader*

The biggest difficulty is changing your own embedded practices. Teaching needs to be adaptive and we need to evolve and

improve constantly. In the beginning I found it difficult to squash my bad habits such as 'hands up' and 'you're correct, move on' style. Now that I have been making a conscious effort and am consistently using *Talk Moves*, it has become more natural.

– Olivia Penhalluriack, *Year 2 Level Leader*

### What changes have you noticed in your classroom since introducing *Talk Moves*?

I have noticed my students grow in confidence with communicating their ideas and strategies. Each student uses *Talk Moves* differently. The quiet students show me through the hand signals where they are up to in the discussion or if they would like to contribute. The high achieving students are using hand signals to show they agree with a fellow class member and they would like to add another strategy they know to further the discussion. *Talk Moves* has opened up a new aspect of my teaching where I can look and formatively assess my students learning through discussion. *Talk Moves* are easily incorporated into my teaching and my students are excited to use them, including in their literacy discussions!

– Nikki Dymond, *Year 1 Level Leader*.

I have noticed the culture of my classroom change. Before using *Talk Moves*, it was mostly my extension students answering questions and the rest wouldn't engage. Now every student 'has a go' and is engaged in the discussion. All strategies are acceptable, from 'counting on' to 'compensation', and varies depending on the individual student.

– Olivia Penhalluriack, *Year 2 Level Leader*

I'm a lot more purposeful about the questions I ask and how I lead mathematical discussions. Students have been very receptive and I think it helped them slow their thinking down, especially in relation to solving complex problems.

– Kylie Smith, *Learning Specialist*

My students are talking more and engaging more in mathematics discussion. They challenge each other's ideas. They share their ideas with each other more readily, because they all know that they could be called on. They are also more willing to have a go because it is a safe environment where their ideas could be challenged and they are allowed to change their thinking.

– Taryn Smith, *Year 6 Level Leader*

# MYTH-BUSTING MATHEMATICS

Kate Smith-Miles - Professor of Applied Mathematics, The University of Melbourne



Check out Kate's YouTube talk on Myth-busting Mathematics, available at <https://youtu.be/lqsDI2CaRg4>

## CHALLENGING STUDENT PERCEPTIONS THAT MATHS IS IRRELEVANT, BORING AND TOO HARD

For mathematics teachers trying to inspire their students to engage and continue with mathematics, there are numerous challenges. Student attitudes towards mathematics are formed early<sup>1</sup>, and it is sometimes hard to undo deeply engrained perceptions that maths is too hard, boring, and irrelevant to their future. But these perception issues must be challenged to ensure each student has equal opportunity for a strong career in a future workforce where high level numerical skills and analytical thinking will be essential and assumed<sup>2</sup>.

Calling out these perceptions as myths, and being equipped with arguments and counter-examples, is the best response for teachers at the front-line. My YouTube video *Myth-busting Mathematics*<sup>3</sup> may provide some ideas for responding to such student perceptions in your classroom. Fundamentally, the belief that mathematics is irrelevant, boring, and a career path that is only suitable for brainy geniuses (and many other misconceptions) often stems from two main causes:

- Lack of understanding of what mathematics really is, and therefore what it can do.
- A belief that some people have a 'maths brain' and others do not.

We should start by acknowledging that most people do not understand *what mathematics really is*. They *think* they know because they have been studying maths since before kindergarten when they learned basic ideas about numbers. Surely after more than a decade of studying a subject, you would be entitled to think you have a clear idea of what it is all about! Unfortunately, the foundational nature of the mathematics studied at school leads to a belief that mathematics is mostly useful for trivial 'everyday' applications, like making recipes using ratios, laying brick patterns using geometry, or figuring out how to place a ladder against a wall.

Certainly this message - that maths is important for everyday activities - is important for raising national numeracy awareness (although it is often counteracted by parents who unhelpfully point out 'I hated maths at school, dropped it as soon as I could, and have never needed it since!'). But we need to be careful that students

don't believe that everyday applications of maths is *all* that mathematics can do.

Just as musical scales are preparation for playing great musical works, and grammar provides the foundations for literature, the kind of *mathematics learned at school is foundational* and mere preparation for something more powerful that most students do not ever see. How do we expose students to the real power and beauty of more significant mathematics, and inspire them to want to learn more, beyond the basic foundations of numeracy?

The future (well-paid) jobs for the current generation of students will demand greater numeracy skills, problem-solving and critical, analytical and logical thinking taught only by studying higher levels of mathematics. There is a depth and breadth of the field of mathematics that the school curriculum simply can't explore. We need students to realise that more *advanced mathematics is important for tackling truly significant problems* like managing spread of diseases, improving green energy, and designing new products and technologies that will revolutionise our lives (just like mathematics has done for centuries).

The challenges for curriculum design and pedagogy are to ensure the foundations are strengthened for all students (for everyday numeracy) while motivating a greater number of students to explore the more creative side of mathematics required for careers that build upon advanced mathematics training.

The movie *Hidden Figures* provides a great example to enable students to imagine how basic concepts explored at school (like parabolas and projectile motion) could be extended in more advanced form to solve real-world problems of great importance (like ensuring the safe return of astronauts). Certainly, not all school maths topics easily lend themselves to this kind of imagined extension. But students can be reassured that each foundational topic extends through the rich tree of mathematics, to enable some truly critical problems to be tackled. Good summaries of such applications exist (e.g. <https://mathigon.org/applications>), but the simplest answer is this: *mathematics is a language, and we use it to describe our world.* With mathematics we can prove facts and estimate uncertainties. We can mathematically model a system (e.g. traffic networks, the human brain, stem cells, stock markets), then use mathematics to predict what would happen if we make changes, and use more mathematics to decide how to improve the system. The opportunities for positive impact - in the corporate world as well as for social good - are endless for those who speak this powerful language. They need to learn to speak it fluently!

With this approach we may be able to convince students that it's important for more people to study advanced mathematics, but many students are affected by their belief that they don't have a 'maths brain'. Hence we need to tackle that misconception too. Early negative experiences with mathematics can be detrimental to student perceptions of their maths ability, and while this can affect students of all genders, it is particularly prevalent amongst girls, who seem to disengage earlier and in greater numbers than boys<sup>4</sup>. There is no evidence that girls perform more poorly than boys in mathematics<sup>5</sup>, and yet the belief that girls' brains are not as 'hard-wired' for mathematics as boys' brains continues

to persist in our society. Given this myth, it should not be surprising that there is evidence that girls are less confident than boys in their mathematics ability. Research conducted by the Australian Mathematical Sciences Institute has shown that this confidence can be changed with very simple intervention exercises<sup>4</sup>. The impact of 'maths anxiety' (which can be so easily passed down from parent to child) and use of mind-shift thinking to increase confidence should inform strategies around teacher training and influencing parental attitudes.

Even if we convince students that there is no such thing as a 'maths brain' - that all students can be taught to think mathematically and problem solve, just as all students can be taught to read and decipher and understand concepts and messages discussed in literature - we still need to tackle the perception that only super smart people should progress to study higher level mathematics. The Hollywood stereotype of the mathematician is unfortunate indeed: typically a male, (often mentally-ill) genius who isolates themselves for many years to tackle an impossibly difficult maths problem that no-one else in the 'real world' really cares about or can understand (such as the portrait of John Nash in *A Beautiful Mind*). This is not a very attractive aspirational goal for most young people! There is an urgent need to communicate more effectively about what mathematics really is, its significance for the real world, and to showcase role models who are relatable, in order to change student perceptions and attitudes towards mathematics.

I will discuss such myth-busting in my keynote talk at MAV19 in December, along with classroom suggestions to help students see that - far from being a dusty and irrelevant subject - mathematics is an exciting evolving subject. Just as it has throughout history, mathematics evolves whenever society faces a challenge, be it an industrial revolution or wartime crises. Our society is facing many significant challenges over the coming decades, and new mathematics is being developed now to ensure solutions are found to pressing problems in food security, climate change, energy efficiency, traffic management, public health, to name just a few. How do we help students understand how maths

can solve these challenges? That is our challenge as educators. I look forward to discussing this important topic with you all at MAV19.

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Kate will deliver a keynote presentation at MAV's annual conference, MAV19, in December.

The conference will be held at La Trobe University in Bundoora and brings together teachers, academics, policy makers, curriculum experts and resource developers from overseas and across Australia.

The theme for this year's conference is *Making Connections*, visit [www.mavvic.edu.au](http://www.mavvic.edu.au) to view the conference program and to book your ticket.

# TEACHERS INCREASING IMPACT

Leonie Anstey - Educational consultant in instructional leadership and mathematics and numeracy education



Figure 1. Foundation students: A snapshot of evidence.

As an educator over the past 20 years, I find it interesting to consider the most impactful actions that teachers have on our schools and classrooms. It is this consideration that has underpinned my thinking as a classroom teacher, mathematics coach and principal.

## ACTIONS AND BELIEFS

As teachers, we are constantly making decisions to determine what learning experiences, dialogue and questioning to utilise in our classrooms. It is interesting to consider that we view ourselves by our beliefs and others by their actions. It's worth thinking about the idea that others see us by our actions. So, do your actions match your beliefs? For example, if you hold a belief that collaboration is important for learning, however your practice has a basis in teacher talk and worked examples, then there is a conflict in belief versus action. So focussing on a pedagogy of reduced teacher talk may be a practice for you to explore.

When we are unsure of the maths content, pedagogical content (how to teach mathematics) or pedagogies, we can

often feel overwhelmed and find ourselves grabbing at ideas or flipping our teaching stance. However, taking the time to identify your core beliefs in teaching and learning, you will enable yourself to 'pick and stick'. If you believe that learning is an active process where we make sense of concepts to communicate in the 21st century; together with the idea that all people can learn mathematics to high levels if taught well (Boaler, J. 2015) then you will make different decisions to someone who believes mathematics is memorisation of facts.

As educators it is important to look to the research to see what is having the biggest impact in mathematics education and trying it out. For at its essence, research means to **re-search**. How does this look in my classroom context? What does it mean for our learners right now?

## TEACHING TEAMS

The biggest impacts in teaching are directly related to collective teacher efficacy (Hattie et al, 2017). This is great news for us as teachers. To put it in general terms;

it is when teaching teams believe they can make the biggest difference to student learning by what they do. In these types of teams, teachers have good self-efficacy they collect and analysis information on what their learners currently know, what they need to know next (learning trajectory) and how they are mostly likely to make progress. Joyce and Showers (2002) proposed that through structured opportunities for professional development teachers acquire new teaching skills; and that they need support provided in their context to be able to transfer new knowledge and skills into their classroom.

Examine Figure 1. Your team might consider this snapshot of evidence to increase impact. What can you see in this photo? As a teacher team, what might this suggest about their learning:

- Today
- Tomorrow
- Next year
- In 2026? (When these learners complete primary school)

With the cognitive analysis of evidence, teachers design learning experiences based on exploring and connecting ideas. By continually asking the following question 'Where are we (learners) now?', we support purposeful learning.

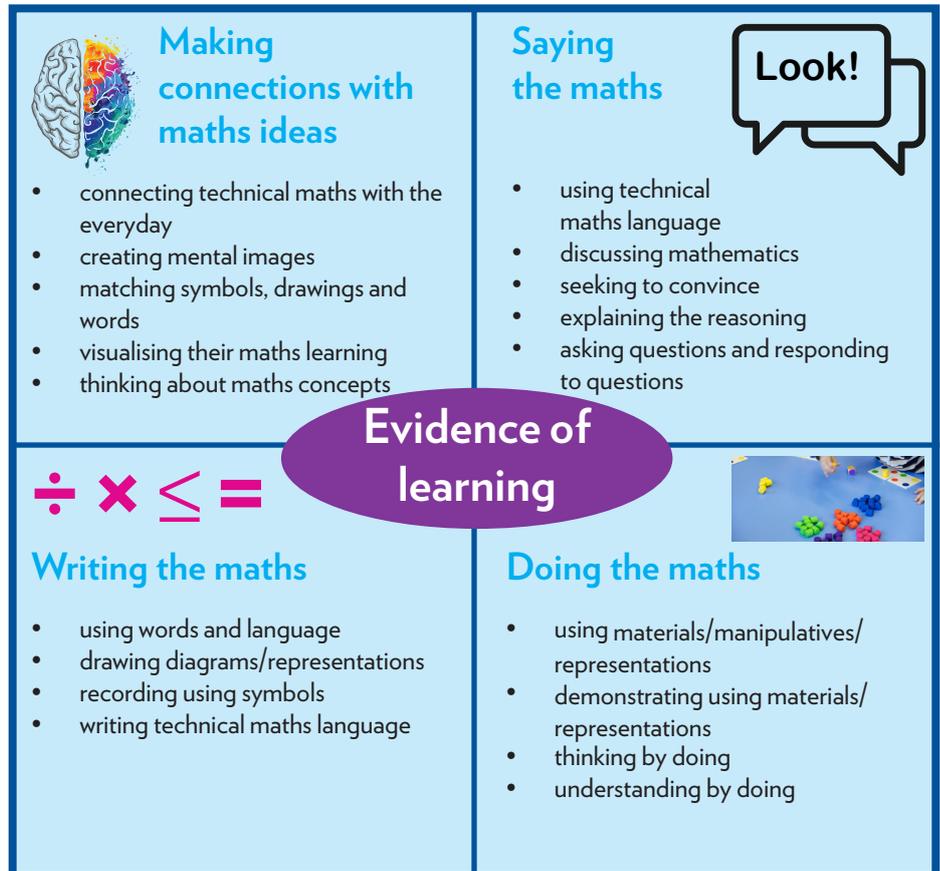
### MAKE, SAY, WRITE, DO

The next important thing for teachers to investigate is their mental model related to their clarity of facilitation. Teacher clarity includes self-reporting grades (how our students see themselves as mathematicians), teacher estimates of achievement (where we place our learners and why), and cognitive task analysis (how we analyse the evidence). In general terms; its teachers knowing what you want your students to understand and what you want students to be able do. Griffin et al (2010) states that there are four types of observable evidence. These are what learners make, say, do and write. Through these four observable behaviours we can infer change in what learners understand, know, feel or think. When we combine what teachers observe with the importance of learners being equally clear about what they must learn and how they can convince themselves and others that they have learnt it we see the development of clear goal setting and progression enabled.

To support teachers and learners to understand the importance of convincing themselves and others, I have developed the Make, Say, Write and Do framework for use with teams, teachers and learners.

In the Make, Say Do, Write framework each of the elements are defined

- **Make:** the learner making connections within and between mathematical concepts
- **Say:** the learner is communicating using mathematical language to discuss, justify and explore their ideas.
- **Do:** the learners are doing the mathematics. They are not repeating the mathematics shown to them or watching the mathematics; they are engaged in the learning experience.
- **Write:** the learner is recording mathematical thinking using language, visuals and symbols while explore and communicate their ideas.



Based on initial research by Griffin et al (2010), and developed by Elevating Learning consultants.

This framework is a useful mental model to observe and plan possible evidence of learning. If the learners haven't made it, said it, done it or written it, then there *is no evidence of learning*. It really supports teachers to move away from statements such as 'I think they can do it, or maybe they just had a bad day'. This framework allows teachers to focus an element of the instructional core defined by Elmore et al (2009), If you can't see it, it is not there.

As teacher teams, we are taking control of what learning happens across classrooms, enabling all students to have opportunity to make progress in their learning. The Make, Say, Do, Write framework supports a team approach to develop learning opportunities to enable learners to develop their conceptual understanding of the mathematics.

It might be useful to recall and reflect on the words of Henry Ford; 'Whether you think you can or you think you can't you're probably right.' As teachers, let's believe and act. We are the difference for student learning.

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# AWESOME WEEKEND AWAY

Paul Tuchten - Year 4 and primary mathematics co-ordinator, Balcombe Grammar School

## TIME DURATION, FINANCIAL MATHEMATICS DATA COLLECTION AND REPRESENTATION

Having just come back from school holidays, the class discussed all the places they had been and children commented if they had been there before. Children are more likely to engage in conversation if they can make a connection, especially if it is a real world experience

The brief for this rich learning activity is to create a timetable for a weekend away to a destination of your choice, price it and then through data collection and representation, compare aspects of your trip to others.

### LESSON 1: PLANNING

**Step 1:** Choose a location in Australia that you would like to visit. Some children chose popular interstate tourist destinations such as Gold Coast and Uluru while others chose places closer to home such as Phillip Island and Mount Buller.

**Step 2a:** Using Webjet or similar sites, find the best flight to your destination. As we are not financially restricted in this activity (although this could be incorporated), children quickly worked out that the best flight would be with no stops and one that is early in the morning to maximise their time away.

**Step 2b:** If you are not flying, then you would be driving or catching public transportation. Children used Google Maps to find the time it would take to get there. If you need to drive from their airport, this will need to be incorporated.

**Step 3:** List all the activities you will be doing there. Ideas ranged from theme parks, mountain bike riding, skiing, to massage and day spa. How long will you be doing this activity for? Interesting to note that some children said they would spend an hour at a theme park. This was a great mini teaching moment for children to discuss that you would most likely spend a whole day there as they are an expensive day out!

**Step 4:** Don't forget that you will need to eat and sleep. How long will you do these for? Where will you sleep?

### LESSON 2: CREATING A TIMETABLE

Using yesterday planning, turn this into a table that shows when you will be doing each activity and its duration. Also keep a record of how much each activity will cost. The conversation around this become quite interesting as children discussed that there is a gap in time from when you land at the airport to when you pick up your hire car. Do we have to allow for our flight home? etc. Doing this, it allows for differentiation as the children need to justify their timings

### LESSON 3: DATA COLLECTION

As a group, we developed some question we would like to know about the whole class. How long will the whole class spend in the air? How much time will people be at amusement parks? How much time will be spent sleeping? Each child had to choose a question that they could survey the class. A key teaching aspect was how to organise a survey to ensure you asked everyone in the class and how we can round time to its nearest hour.

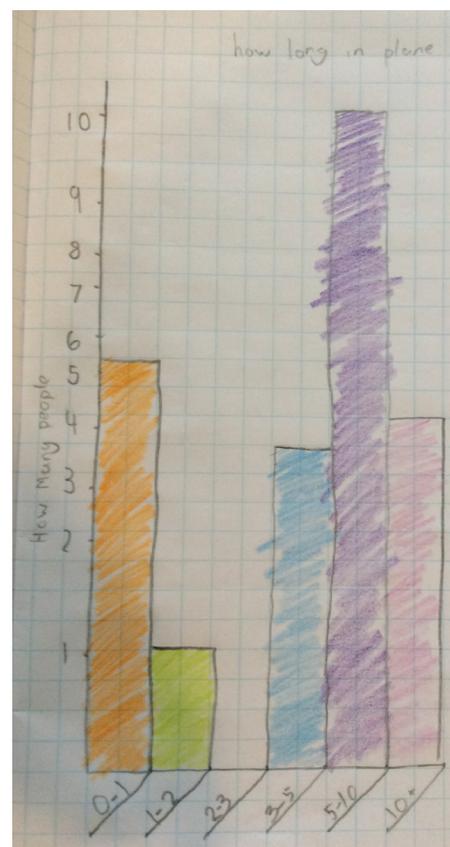
### LESSON 4: DATA REPRESENTATION

As we bring this fantastic lesson to a close, children were then taught how to tally then create a bar graph of yesterday's findings and summarise what they had found out.

0-1		5
1-2		1
2-3		0
3-5		4
5-10		10
10+		4

### SUMMARY

This was a really engaging activity. The children could connect with it as they either went to a place they had been or planned a trip to somewhere they would like to go to. Children had the opportunity to learn more about places in Australia and share their experiences. By working in pairs, they had to compromise as well as teach each other.

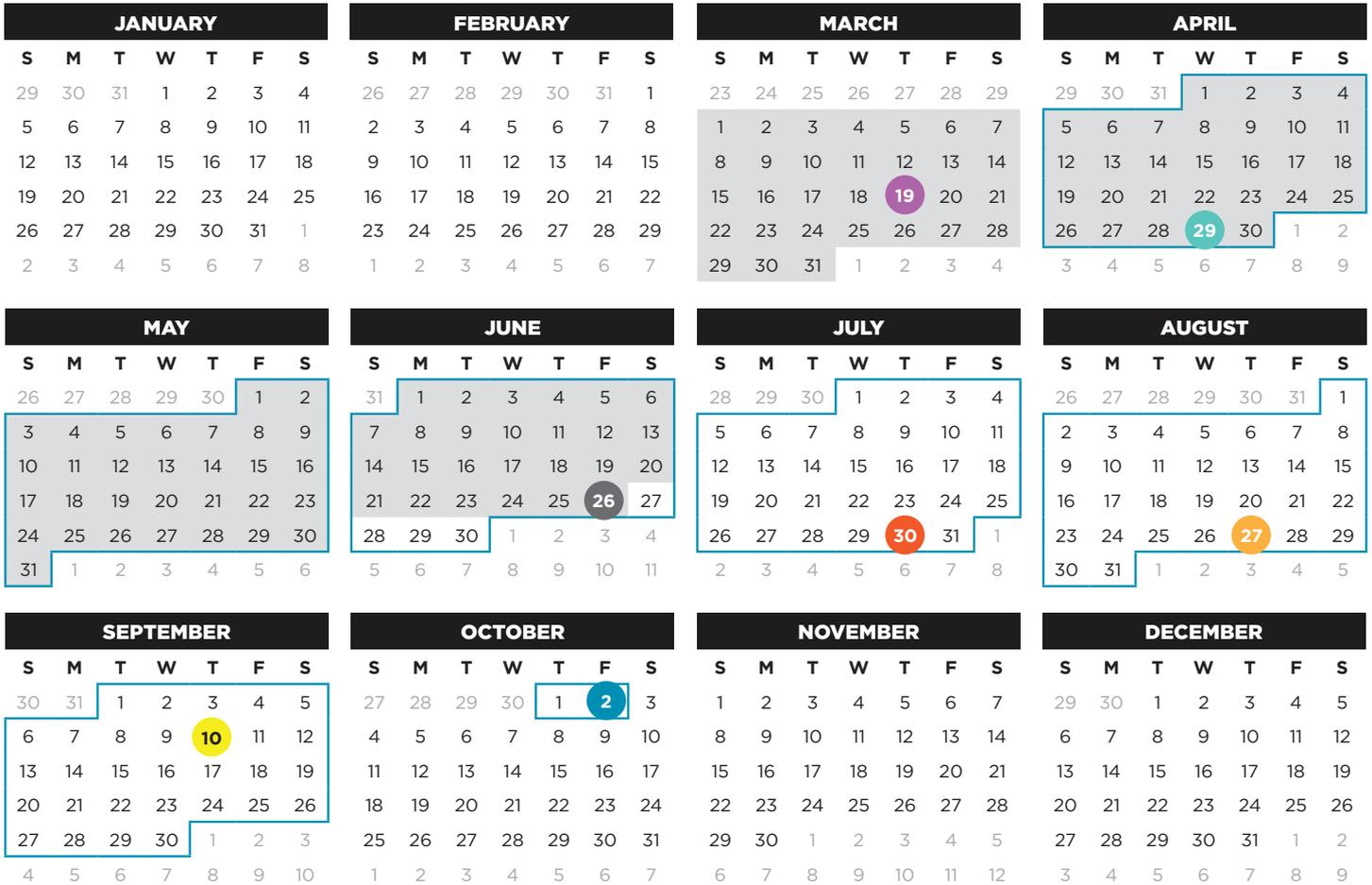


### VICTORIAN CURRICULUM OUTCOMES

- Use am and pm notation and solve simple time problems, calculating duration of time.
- Create and solve number story problems in addition and subtraction that are appropriate to the student level of number understanding using real life situations. Use measurement and money where possible. Record these as written story problems and related numerical equations.
- Select and trial methods for data collection, including survey questions and recording sheets.
- Construct suitable data displays, with and without the use of digital technologies, from given or collected data. Include tables, column graphs and picture graphs where one picture can represent many data values.

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-  Maths Enrichment | April to October Results due 2 October
-  Australian Intermediate Mathematics Olympiad | Thursday 10 September



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# RECIPROCAL TEACHING IN MATHS

Yvonne Reilly and Jodie Parsons - Sunshine College

Ask any maths teacher to articulate one of their most frustrating moments and it probably goes something like this, 'I've seen <student> answer this type of question many times in class, however when I go through their book or analyse their results in a standardised test, they get it wrong.'

We as teachers know it is not uncommon to see students perform tasks with a high degree of mastery, to solve a question with the correct mathematical procedures but when faced with a worded-question their answers are often incorrect. They appear to have either not properly understood or inadequately addressed the question. Some students get lost in the literacy of the question, tied up with confusion about which tasks to perform first and which pieces of information to ignore and which pieces to extract. In mathematics, worded-questions present a number of challenges for students, before they can solve the question (with the appropriate mathematics) they must first understand the question. They must navigate the challenges of mathematical literacy.

When solving a worded-question students go through a number of processes. They must first make sense of the language of the problem, then they are required to decode the mathematical English of the problem. For example, words such as 'difference', 'product' and 'volume' have an explicit meaning in mathematics. Next students are faced with the need to decide the type of mathematics they must use and then interpret or possibly prioritise the order in which the mathematics to be used. For example, 'take 5 from 7' requires students to understand that the computation is 7 subtract 5 which does not follow the syntax of the question or to add on from the five which is opposite the accepted meaning of 'take from'. And finally, if students manage to decode and comprehend the everyday language of the problem, they then need a solid grasp of the academic vocabulary, words such as perimeter and trapezium.

In our early attempts to improve the ability of students to answer worded-problems we targeted their understanding of academic vocabulary, following the advice of experts such as Marzano, but we soon found that we were not seeing the level of student improvement we needed.

We realised we needed to build comprehension of the mathematical literacy of worded problems with our students, to build these skills we adopted the well-known literacy strategy of Reciprocal Teaching.

Reciprocal Teaching is an instructional approach whereby students work in small groups to predict what the text is about, clarify any words or phrases unknown to them, and identify any questions which spring from reading the text and finally summarising the main idea of the text. If you would like to read more about this literacy strategy we recommend the paper by Palincsar & Brown (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction*, 1, 117-175 [https://people.ucsc.edu/~gwells/Files/Courses\\_Folder/ED%20261%20Papers/Palincsar%20Reciprocal%20Teaching.pdf](https://people.ucsc.edu/~gwells/Files/Courses_Folder/ED%20261%20Papers/Palincsar%20Reciprocal%20Teaching.pdf).

For this strategy to be applied effectively in mathematics we had to adapt it to allow students to identify the 'big question', which is either specified or implied in worded-problems and then solve it. We also modified the summarise section to provide students with an opportunity to reflect on their learning.

## MATHS FUTURES PROGRAM

Sunshine College includes one 50-minute period of reciprocal teaching each week as part of our Maths Futures Program.

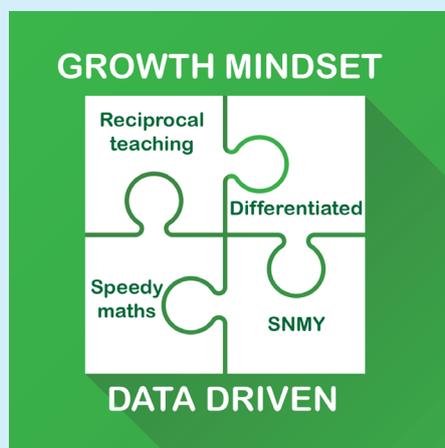


Figure 1. Maths Futures program, Sunshine College.

In order to maximise the effectiveness of this strategy in mathematics each reciprocal teaching lesson is taught around the following instructional principles.

1. Reciprocal teaching is entirely collaborative process and therefore we encourage all students to work in groups of three. We have found this group size to be big enough to bring different perspectives to the discussions but small enough to ensure everyone is contributing. Students collaborate throughout the process. There are no group leaders and each student in the group works at a similar cognitive speed and maths level as one another.
2. To ensure students get the maximum benefit of the reciprocal teaching part of our program we provide students with both a template to complete and a prompt board. Students work through the process together to build meaning and understanding. If a group can solve a problem without the need of the process then they have chosen a question which is too easy for them (see Figure 2).
3. Each worded-problem is fit for purpose. That is we create problems across multiple levels which allow students to access both the language and the maths at a level which is just right for their learning. In our classes we offer six different levels. The higher levelled questions are not just about more difficult maths; they are about more difficult or dense text.
4. The collection of problems offered span all strands of mathematics and not just topics which match the current syllabus of our differentiated lessons. This means that students are constantly exposed to all the topics of mathematics throughout a year.
5. To manage the lesson most effectively and to ensure all students in the group have access to the problem we print out three identical copies of each question, laminate them and store them in an envelope with a copy of the problem pasted on the front. Students select a question from a large selection of questions and work collaboratively to solve it. Each worded-problem is formatted with pictures or graphics which will help with the initial prediction of problem content. An example of how we format our problems can be seen in Figure 3.
6. Lastly, we provide sufficient time for students to complete this process so that it impacts their learning.

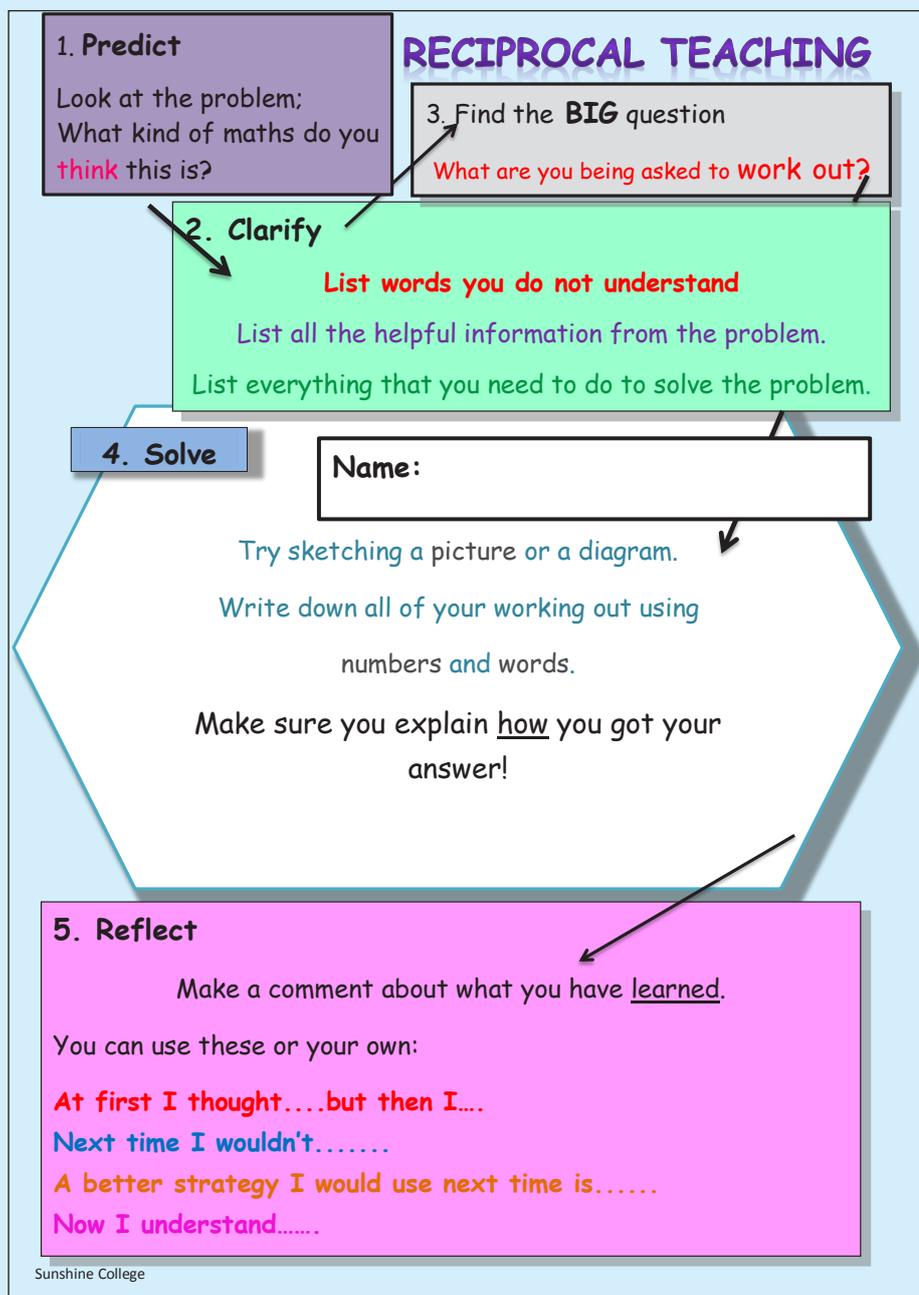


Figure 2. Prompt board.

**TRAFFIC LIGHTS**

A set of traffic lights is red for half the time, orange for  $\frac{1}{10}$  of the time and green for the rest of the time.

For what fraction of the time is the set of traffic lights on green?



Figure 3. An example of a Reciprocal Teaching problem, Sunshine College.

This means that each group normally completes only two or three worded-problems per lesson. This may concern some teachers who would be happier with students completing far more examples in a lesson, but remember the purpose of this process is not to get better at the maths per se, but to get better at accessing and comprehending the maths. If students are completely questions much quicker this possibly demonstrates they are choosing questions which are too easy for their group.

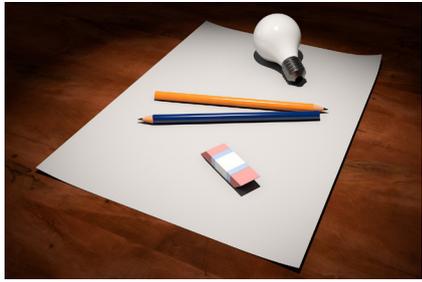
If you would like to trial this approach in your own class we recommend using problems from past NAPLAN papers as a starting point – but remove the multiple choice option. Creating your own questions is a time consuming process, a bit of time is required to ensure they look appealing and to come up with a process of labelling them that helps students to choose problems which are just right for them.

Sunshine College has a bank of 250+ problems which span six levels to share with anyone interested in trialling reciprocal teaching in their own classes. If you are interested in doing so please contact the authors: reilly.yvonne.c@edumail.vic.gov.au or parsons.jodie.m@edumail.vic.gov.au.

# PUZZLES

Michael Nelson - Learning specialist, Drysdale Primary School

## LOWER PRIMARY



Our teacher told us to draw a four sided shape. I drew one and so did my friend, but hers had sides that had different lengths to mine. Our teacher told us that we had both drawn the same shape. How is this possible?

*Recognise and represent division as grouping into equal sets and solve simple problems using these representations. (VCMNA109)*

## MIDDLE PRIMARY



I was going on a trip that was 36km long. The distance I had travelled so far is half of the distance that I have left. How far have I travelled?

*Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies. (VCMNA160)*

## UPPER PRIMARY



I took the number 73, turned it around to get 37 and then split it into 3 and 7. All 4 are prime. How many other numbers can I do this with?

*Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction. (VCMNA192)*



My friend wrote down a three digit number. I couldn't see what he had written but he told me that it had 12 ones. What numbers could he have written down?

*Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting. (VCMNA105)*



Nick has nine cards, all of which are different. He divides the cards into three equal groups, with the sum of each group equaling 15. How many different ways can he divide his cards?

*Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation. (VCMNA133)*

$$\begin{array}{r} 24 + \\ \hline 26 \end{array}$$

How many numbers can be made by adding two consecutive positive integers together?

*Investigate everyday situations that use integers. Locate and represent these numbers on a number line. (VCMNA210)*

Images from Pixabay (L-R on top row): Arek Socha, Ryan McGuire.  
(L-R bottom row): \_Alicja\_, tomekwalecki

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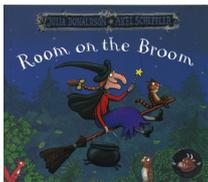
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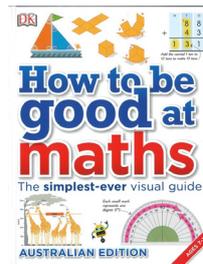


## ROOM ON THE BROOM

F-3

The witch and her cat fly happily over forests, rivers and mountains on their broomstick until a stormy wind blows away the witch's hat, bow and wand. Luckily, they are retrieved by a dog, a bird and a frog, who are all keen for a ride on the broom. It's a case of the more, the merrier, but the broomstick isn't used to such a heavy load and it's not long before... SNAP! It breaks in two! And with a greedy dragon looking for a snack, the witch's animal pals better think fast. A very funny story of quick wits and friendship.

\$13.64 (MEMBER)  
\$17.05 (NON MEMBER)

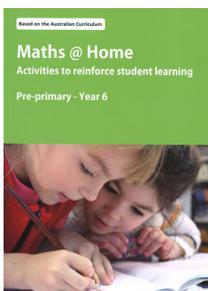


## HOW TO BE GOOD AT MATHS

3-9

The unique visual approach of *How to be Good at Maths* makes basic maths easier to understand than ever before, with short, simple explanations that demystify even the most challenging topics. Find out how much you would weigh on Jupiter, calculate the average age of your football team and even use pizza to understand pesky fractions. Unlike other maths workbooks, *How to be Good at Maths* introduces each topic with colourful pictures, real-life examples and fascinating facts. Making maths fun and easy, it is ideal for reluctant mathematicians or for revising before a test.

\$29.58 (MEMBER)  
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## MATHS @ HOME

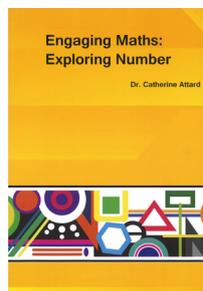
F-6

Parents often go to great lengths to help their children succeed in their education. Unfortunately many parents believe that helping their children with mathematics is beyond their abilities. For some, learning mathematics was a dull and uninspiring experience that they are not keen to revisit. For others, the mention of mathematics is associated with a profound sense of fear or anxiety.

Fortunately a rapidly growing body of scientific research has led educators to question much of what they thought they knew about the teaching and learning of mathematics. Teachers and parents now have a shared responsibility to ensure that in future students can learn mathematics with confidence and understanding.

This Parent Guide was developed by the Mathematical Association of Western Australia. It is designed to assist parents who wish to ensure that their children are capable and confident users of mathematics.

\$11.05 (MEMBER)  
\$13.81 (NON MEMBER)



## ENGAGING MATHS: EXPLORING NUMBER

F-6

This book presents a collection of 10 rich activities that address aspects of the Number and Algebra strand of the Australian Curriculum: Mathematics. More importantly, the activities are underpinned by the processes of mathematics described in the proficiency strands of the curriculum. Each of the activities is supported by detailed ideas for implementation, reflection, assessment, differentiation and curriculum links.

Something that sets this book apart from other teacher resource books is that its prime purpose is to enhance the teaching of mathematics rather than focusing purely on learning. By using this book as a professional learning tool, not only will your teaching be enhanced, your students' learning and engagement will also benefit as a result. It is hoped that the structure of the book will help you, the teacher, to reflect upon your current practices and find ways of adapting the things that already happen each day in your mathematics lessons. The book includes resource lists, curriculum links, implementation ideas, reflection starters, ideas for differentiation and black line masters for students.

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