THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR 3/16

GAME ON

Games are a wonderful way to introduce mathematical concepts in the classroom and the home.

INSIDE

I love a game of cards. I have wonderful memories of playing with my grandparents and now take great joy in watching my own children play with their grandparents. A little healthy competition is great in so many ways and fabulous in helping to build mathematical content knowledge and the proficiency strands understanding, fluency, problem solving and reasoning.

Games are relevant to both classroom and home, and while both contexts will have a different focus and reasoning behind playing games, the benefits are similar. Games allow students to make meaningful connections with the mathematics they have learnt. Some benefits are:

Fun and engaging - let's admit it most of us enjoying playing games, especially if we like the people we are playing against. A little competition is fun.

Skill practice - the opportunity for practice is high, allowing students to practice their learning in a non threatening environment.

Continued on page 4

FROM THE PRESIDENT

Jim Spithill - ACER

THE COMMON DENOMINATOR
The MAV's magazine published for its members.

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It is now commonplace to refer to 'disruption' in so many areas of civil society. In a recent piece at The Conversation, the Vice-Chancellor of Western Sydney University, Barney Glover, was quoted: 'Disruption is not a distant rumble. It is upon us.' The rapid churn in the types of jobs for the future poses a real challenge to educators: Education for what?

And it is not just in education: think Uber, AirBnB, robotics, Aldi vs the supermarkets duopoly, the decline of Australia Post and of print newspapers. There is a sense about that everything old is up to be challenged. How much longer can we maintain an essentially 19th century industrial model of how to structure education?

For the MAV the immediate challenge is to recruit a new CEO to follow on the many years of outstanding service and leadership from Simon Pryor. The position description encompasses the expectations and opportunities that reside in the role, and we are confident that we will find the right person to take the reins. Stay tuned to MAVlist for the latest developments.

And yet, amidst all the change, some things remain the same. I have been reading the updated edition of *Gough Whitlam HIS TIME* by Jenny Hocking. What were the hot button issues in the first year of that government in 1973? 'The grand and unifying theme was equality of opportunity, and at the heart of equality of opportunity lay education ... dramatic disparities of educational opportunity between private and government schools were ... morally unjust and socially wasteful.'

By the time you read this a federal election will have been run and won. Two competing views of education were in play: education as a leveller of the playing field versus education as a means of perpetuating inequality (to mention Bourdieu). I wonder which one prevailed?

The 2016–2017 MAV Council has been bolstered by the inclusion of several new members who will provide fresh thinking about such matters as Early Years Numeracy, the VCAL sector and better ways of communicating with the membership. The months ahead see the confluence of a new CEO and the development of a new strategic plan.

How to plan in a disruptive environment is a real challenge. There is more to these developments than mere following of process.

The MAV is no more than the sum of its membership. Recently on MAVlist there was an advertisement for an Outreach Officer for the ReSolve: Maths by Inquiry project in Victoria. Inside a week there were dozens of applications for what is a temporary role within the prescribed time frame of the project. What leapt from the page in reading the expressions of interest was the number of people in the mathematics education community who have a passion for improving students' learning.

There is hope for the future, however it may evolve!

REFERENCES

http://theconversation.com/we-cant-have-a-strong-economy-without-a-strong-university-sector-warns-vc-59450

Gough Whitlam HIS TIME by Jenny Hocking p85

https://en.wikipedia.org/wiki/Cultural_reproduction

In July, MAV welcomes Digital Learning and Teaching Victoria as a tenant at Cliveden in Brunswick. With the closure of the Statewide Resource Centre in Carlton, DLTV was looking for a new home, and MAV was delighted to be able to offer them the space they needed. Having DLTV in the same building will provide the two associations with plenty of opportunities to discuss current trends in both maths education and digital learning. We hope that this will result in some joint activities in the future and look forward to a fruitful relationship.

Pictured above: Sally Turnbull, Acting CEO with Mordechai Katash, Executive Officer, DLTV.

2016 MAV PD

During 2016 a variety of presenters and MAV's own mathematics educational consultants will present workshops focusing on innovative teaching practice. **Make sure you reserve a place by booking online early, www.mav.vic.edu.au/pd.**

TOPIC	DATE	YEARS	PRESENTER
Using SCRATCH as a programming tool in the mathematics classroom (part 1)	13/7/16	6 - 9	Jennifer Palisse
2016 Making confident mathematics teachers: teaching junior secondary mathematics - Non-linear algebra	16/7/16	7 - 10	Ian Lowe
Engaging maths games to build skills, confidence and higher order thinking with the Think Square	20/7/16	F - 9	Andrew Lorimer-Derham
Using SCRATCH as a programming tool in the mathematics classroom (part 2)	27/7/16	6 - 9	Jennifer Palisse
Waging war on worksheets	28/7/16	F - 6	Martin Holt
Tune me in, short sharp maths warm-ups to get your lessons rolling	4/8/16	F - 6	Tim Colman
Engaging maths games to build skills, confidence and higher order thinking with the Think Square	16/8/16	5 - 10	Andrew Lorimer-Derham
Using rich tasks and open-ended questions to develop mathematical proficiency	18/8/16	F - 6	Jen Briggs and Kate Osbourne
Running an effective family maths night	31/8/16	F - 10	Helen Haralambous
2016 Making confident mathematics teachers: teaching junior secondary mathematics - Introduction and measurement	10/9/16	7 - 10	Ian Lowe
Thought provoking mathematics through Scootle	13/9/16	F - 10	Martin Richards and Jennifer Bowden
2016 Making confident mathematics teachers: teaching junior secondary mathematics - Geometry	8/10/16	7 - 10	Ian Lowe
Working mathematically in the early years	11/10/16	EY	Doug Williams
What's the story?	20/10/16	F - 6	Ellen Corovic
2016 Making confident mathematics teachers: teaching junior secondary mathematics - Statistics and probability	22/10/16	7 - 10	Ian Lowe
Coding - a hackathon for beginners	25/10/16	F - 6	Sarah Longhurst
MAV conference	1/12/16 2/12/16	EY - VCE	Various

SOAK UP MATHS THIS SUMMER

APPLICATIONS FOR THE 2017 NATIONAL MATHEMATICS SUMMER SCHOOL ARE NOW OPEN.

Being awarded a place at the National Mathematics Summer School gives students who love mathematics an opportunity to have fun together as they discuss and explore mathematics from some very different perspectives. Give your students who love mathematics a chance to apply. As there is generally a limit of two applicants from each school being selected, this gives more chance for a variety of schools to be involved.

Students from all over Australia who are in Year 11 in 2016 apply. It is a two-week program held at the Australian National University. Many enduring friendships are made on this very special summer school, now in its 49th year.

There is a strong emphasis on solving problems and many students often find themselves working on problems well into the night (because they want to)! There is also recreation, exploration and fun and many activities, tours and talks are organised. During afternoon recreation, students are free to socialise and explore Canberra (popular activities include ice skating, ten pin bowling, tandem bike riding around the lake). There is an entertaining dinner and concert on the last night.

Selection is based on the strength of a student's written application, a teacher reference, and the creativity and perseverance you show on a problem solving test. The NMSS contribution is $1100 and a travel surcharge will apply if students/parents want NMSS to organise return travel from Melbourne airport.

The 2017 NMSS will be held in Canberra from 8-21 January.

The closing date for applications is 29 July, 2016 and the 90-minute selection MAV Test for NMSS will be held on 5 August, 2016 at the applicant's school.

Application form can be downloaded at www.mav.vic.edu.au/activities

Further information is available on the NMSS website: www.nmss.edu.au

GAME ON!

Jennifer Bowden - Mathematical Education Consultant, MAV

Differentiation - a simple game such as *Snap* can be modified for different skill levels. Children often choose games that challenge them.

Assessment - when children are absorbed in a board, dice or card game, teachers can observe thinking strategies and understanding.

Connections - games are an effective way to make connections between home and school. Giving students a game to play a homework allows parents to participate in their children's education.

Games teach students import social and life skills such as collaboration and communication. Students develop persistence, fair play, game etiquette and humility.

HINTS FOR SUCCESSFUL CLASSROOM GAMES

Alridge & Badham (1993)

- Make sure the game matches the mathematical objective
- Use games for specific purposes, not just time-fillers
- Keep the number of players from 2-4, so that turns come around quickly
- The game should have enough of an element of chance so that it allows weaker students to feel that they have chance of winning
- Keep the game completion time short
- Use 5-6 basic game structures so children become familiar with the rules - vary the mathematics not the rules
- Send an established game home with a child for homework
- Invite children to create their own board games or variations of games.

There are many ways your students can engage in games such as classroom warm ups, small group lessons, classroom investigations, homework activities, Family Maths Nights or even a whole school or MAV State Wide Games Days.

The online gaming world is becoming more accessible to classroom teachers, this has become a particular focus since coding has entered out curriculum. Ensuring this practice is completed with good pedagogy, including collaboration and communication and along with the tips mentioned previously is essential for successful implementation.

It can be tricky to find outstanding resources and innovative ideas about how to incorporate games into your classroom.

Some of my favourite are:

- Scootle
- Nrich
- Dr Paul Swan
- Calculating Changes
- Maths 300

REFERENCES

Aldridge, S. & Badham, V. (1993). Beyond just a game. *Pamphlet Number 21* . Primary Mathematics Association.

http://nrich.maths.org/2489

For more information about how you can use games to enhance your classroom practice or to have a MAV Education Consultant visit your school to share some engaging practices in using games or to facilitate a Family Maths Night email Jen Bowden jbowden@mav.vic.edu.au.

You are welcome to join to Jen Bowden and Ellen Corovic's presentation at The Education Show (2-4 September 2016), *Game On! Engaging students through games and making connection with home.* www.theeducationshow.com.au.

2016 MAV CONFERENCE

1 AND 2 DECEMBER 2016
LA TROBE UNIVERSITY BUNDOORA

For the first time we are offering an early bird rate to attend the conference. Register before the 1 August 2016 to receive a substantial discount on registration prices.

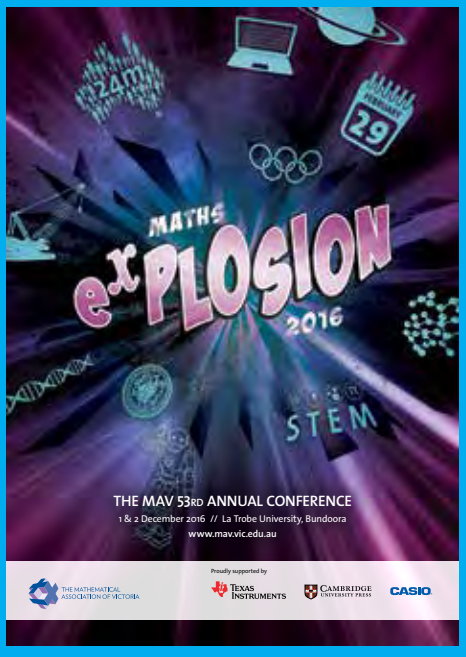
We are trying a new format this year. Instead of having keynote speakers in each session we will be running a keynote series at the beginning of each day.

Six keynotes will present in different theatres on a variety of subjects and year levels. Hopefully this will mean you get to see the keynote you want to on the day.

If you have any questions please contact Julie Allen, jallen@mav.vic.edu.au or telephone 03 9380 2399.

	Early bird	After 1/8/16
Member Metro: one day	$258	$284
Member Metro: two days	$516	$567
Member Non-Metro: one day	$250	$275
Member Non-Metro: two days	$508	$559
Non Member: one day	$337	$371
Non Member: two days	$675	$742
Student: one day	$133	$146
Student: two days	$266	$293

Register for the conference now: www.mav.vic.edu.au.



GREAT GAMES

Louise Gray - parent and editor, *The Common Denominator*

My second child is great game player. He loves almost every game we have and can spend hours playing games with willing opponents. Our deck of *Uno* cards gets a regular workout.

Consequently, our games cupboard is bulging and I made a rule: no more games! I broke that rule a few weeks after I made it - and I'm so glad I did. I want to share a few of our favourites with you.

QWIRKLE

This game has won a bunch of awards and its easy to see why. *Qwirkle* consists of 108 wooden tiles. Each tile has a shape on it, with six different shapes in six different colors, the aim is to lay the tiles and make a Qwirkle.

A Qwirkle is a line of either all six shapes or colors. Scoring is simple and the rules are quickly learnt by players of any age. I've successfully played with 4 - 70 year olds. The game can be played on many levels. Adults will seek out strategic moves and kids may at first concentrate on laying their tiles, and advance to strategy becoming part of the play.

RAT A TAT CAT

Rat a Tat Cat is a super card game. It's premise is quite simple - the player with the lowest score wins.

Each player is dealt four card face down. The player can look at the first and fourth card but the middle two cards remain secret. Players take turns drawing a card from the middle pile and they can elect to switch that card with their four dealt cards in order to obtain the lowest score. There are a couple of surprise cards, including the 'Peek' card which allows a player to take a peek at their second or third card. The game ends when a player feels that they have the lowest score. They knock and say 'Rat a Tat Cat'. Then each player gets a final turn before showing their cards and adding up their score.

There is lots of mathematics in this game. Players need to think about the probability of getting high and low cards, adding up their scores, making judgements about the strategy of others and remembering which cards they have.

My eldest son came up with a terrific extension to this game.

Qwirkle is a game easily understood by players of all ages.

Instead of adding up the total number of the four cards, he suggested that the four piles represent thousands, hundreds, tens and units. All of a sudden, drawing a 9 wasn't so bad! As long as it was placed in the units column of course.

We have played around with a few rule modifications that focus on extending mathematical abilities and encouraging the children to try and think a little more laterally. This is a solid family game that is portable. A good one to take on holidays, trust me - you'll play this for hours!

SLEEPING QUEENS

This card game came highly recommended to me and I know why. It's addictive. The aim is to wake up the sleeping queen cards. The rules are a little complicated to explain but once a few hands are played, it's quite simple. This game was devised by a clever six year old child. It contains lots of fun elements that appeal to children: potions, dragons, knights and queens.

The mathematical element is strong. Players need to discard cards at each turn. They can discard based on their ability to come up with an equation.

For example, if a player has a 1, 5, 6, 7 and 8 in their hand, they could discard the 1, 5 and 6 during one turn because 1 + 5 = 6.

As our family became more confident in the game, we modified the rules to include negative numbers, subtraction, multiplication and division. Order of operations has been a good talking point with our new rule modifications.

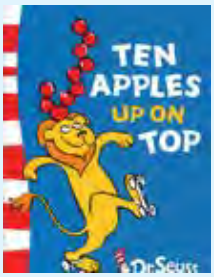
As an example, if a player had a 3, 2, 5, 6 and 1 in their hand they could get rid of all five cards: 3 x 2 = 6 - 5 = 1.

It's great to see how creative the kids get with their equations - of course, the more creative you are - the more cards you can get rid of. I've been surprised how eager my 6 and 9 year old are to play hand after hand. It's great to see them engaging with these new concepts in a fun way.

Is there a particular game, book or toy that inspires mathematical thinking in your home or classroom? Email ecorovic@mav.vic.edu.au to share your story in the next edition of *The Common Denominator*.

10 APPLES - AN INVESTIGATION

Tracy Henderson - Foundation teacher, Serpell Primary School



INVESTIGATING DR SEUSS' *TEN APPLES UP ON TOP* FOR THE MATH'S TALENT QUEST

Learning to count is made fun in this tale of a dog, a lion and a tiger all showing off how many apples they can balance on their heads as they skip, walk the tightrope and roller skate their way through the book.

The animals try to prevent the apples from falling off their heads and become competitive as the story progresses. The dilemma of trying to balance all the apples up on top is solved by the end. This book is just one in the series developed by Dr Seuss, which incorporates the use of rhyme, rhythm and repetition to engage children and their love of reading.

The zany artwork and humour in *Ten Apples Up On Top* captured the interest of my Foundation class. They were curious to find out how the characters could balance objects up on top of their heads, and so we chose this story for our Maths Talent Quest investigation.

We incorporated a variety of mathematical concepts throughout our investigation, ranging from addition, subtraction, sequencing numbers, patterns, skip counting, volume and fractions. We linked the story to our school philosophy program by following the philosophical structure of inquiry. The students discussed what they liked about the story giving reasons for their responses, they discussed what puzzled them about the story. They made predictions and they formed investigative questions, which were explored and reflected upon throughout each lesson.

The lessons involved investigating what they knew about the number 10, counting and recording apples on top of their heads and exploring all the different ways they could make collections of ten. We investigated the different patterns they could make using red, green and golden apples and worked out what would happen if the apples fell by subtracting different numbers of apples from the number ten.

Students investigated how many apples the apple cart in the story would hold. They constructed apple carts using a variety of materials and tested these to determine how many apples they would hold. We explored how many apples the animals would balance altogether if each of them balanced ten. Students skip counted the total amount of apples and then worked out how many apples our class could balance altogether if they all had ten apples up on top of their heads.

We discovered whole apples were difficult to balance. So we explored the possibility of balancing apples that were cut into different fractions. We cut apples into halves and quarters and attempted to balance these. The children answered yes/no questions to gather information about whether they could balance different fractional parts. They made predictions, attempted to balance the apples and then wrote a reflection based on the results.

The children discovered that the animals would have had difficulty balancing the apples, so they explored other objects that would balance better. They tested a range of other materials to determine which objects were the easiest to balance. They predicted that the apples would be heavy so weighing them would be a great way to test this. They used balance scales to determine how heavy the apples would be.

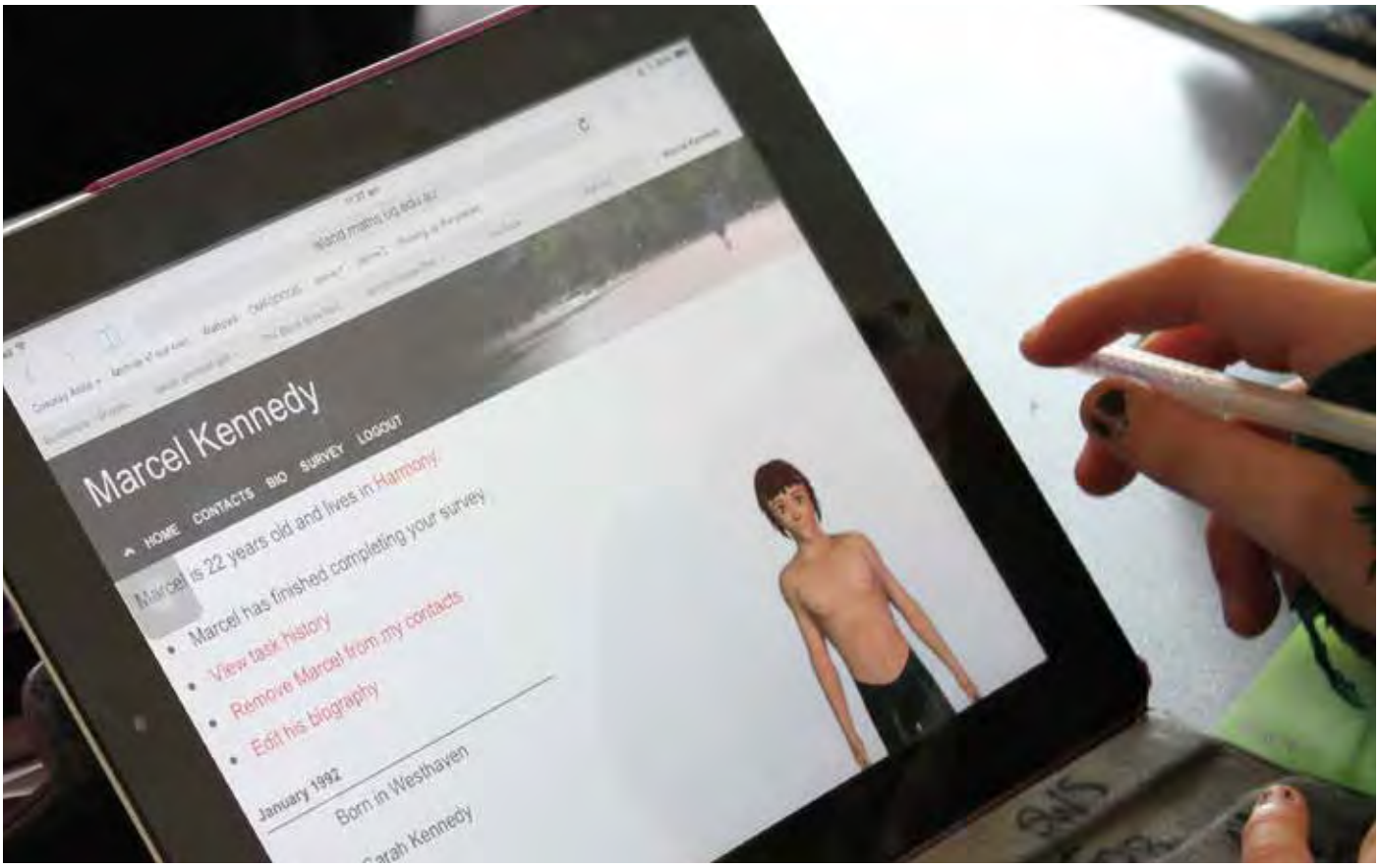
The students wrote personal reflections about what they had discovered throughout the investigation. They came to the conclusion, after all of our experimentation, that it would be impossible to balance ten up on top.

This investigation gave students the realisation that a story can be brought to life with a real investigation and explored from a different perspective using a mathematical focus and a rich learning experience.

Serpell Primary School is a Mathematics Active School, to find out more, visit www.mav.vic.edu.au.

THE ISLANDS IN SCHOOLS

Claire Hart - Project Officer, The Islands Project, RMIT University



Junior secondary students completing the Islands activities in class.

THE ISLANDS – WHAT IS IT?

The Islands in Schools Project set out to improve student attitudes towards, and understanding of, the role of statistics and data analysis in the real world by implementing the Islands in junior secondary classes across Australia.

The Islands is a virtual playground for engaging students in realistic statistical data investigations without the many practical and ethical constraints imposed by real research involving humans.

Students can propose statistical questions, design investigations and collect the necessary data for statistical analysis and interpretation.

The wide range of data and tasks available on the Islands caters to many scientific areas and student interests.

KEY FEATURES OF THE ISLANDS

Over 34,000 virtual inhabitants spread across three geographically diverse islands

- Inhabitants are born, die, relocate, and get sick in advanced time
- Inhabitants have unique personal histories, genetics and ethnicity
- 200+ tasks to experiment with
- Inhabitants can refuse consent, lie, and sleep during the night, so students learn about the practicalities of investigations
- Supports many different types of investigations, including surveys, observational studies, experiments, and interviews
- Also features agricultural, climatic, educational and employment models.

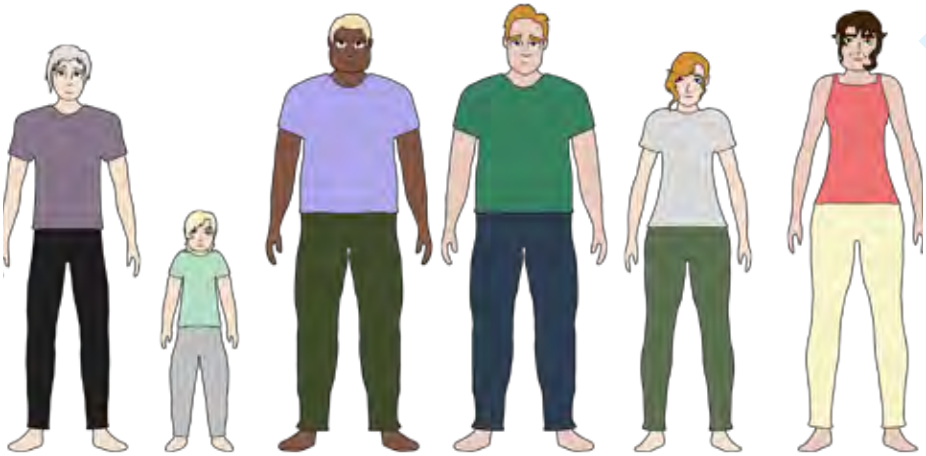
THE ISLANDS LEARNING ACTIVITIES

The Islands learning activities present students with meaningful, contextualised problems to be investigated. The activities cover a broad range of topics, including mental health, exercise performance, lung capacity across age, and death rates. The activities and suggested rubrics are free to download from www.islandsinschools.com.au/teacher-resources.

BENEFITS

The Islands-based learning resources offer a number of benefits to teachers, including:

- Free and quick to sign up for an Islands account
- Free to download the Islands learning resources
- Students can conduct realistic investigations without real-world practical and ethical constraints
- The wide range of data and 200+ tasks available on the Islands cater to many scientific areas and student interests
- Teachers can easily add and manage classes on the Islands.



Each Islander has their own personal story which details their family history, relationships, offspring, and medical history

HOW CAN I USE IT?

The Islands in Schools Project is now complete and teachers from across Australia can visit www.islandsinschools.com.au to sign up for a free Islands account, learn more about using the Islands and download the free learning resources.

Teachers are provided with coordinator rights when they sign-up as a teacher. This enables teachers to have access to information about a group/class, the ability to add users and view user details, change task filters, create new instances, and the details of groups of classes they have added. To read more about coordinator rights, visit www.islandsinschools.com.au/islands-guide/tools.

WHAT ARE TEACHERS SAYING?

'The student engagement was phenomenal. No pupil was off task during our computer lessons.'

'Students were very engaged in the virtual environment. They enjoyed the history of the islanders and were really interested in the types of tests they could do.'

'I liked the fact the students were able to collect their own data. They could pose their own questions and be able to collect data and analyse it to generate conclusions.'

All teachers are welcome to use the Islands resources and are encouraged to share their experiences with others. To learn more about the Islands, contact Dr James Baglin, james.baglin@rmit.edu.au or visit www.islandsinschools.com.au.

MAV YEAR 10 MATHS CAMP

Are you a regional school and teach Year 9 students who are are gifted and talented in mathematics?

In 2017, the MAV will hold a Mathematics Stimulus Camp for Year 10 students in the Term 1 holidays. The camp is part of a state government funded Strategic Partnerships Program. Students gather with like-minded peers and explore hands-on, industry-related problems. The week-long Melbourne camp involves city experiences and problem solving challenges and enjoyable activities. Students work in small groups on a real life open-ended project.

THE 2016 CAMP

2016 saw the inaugural MAV Mathematics Stimulus Camp. Students spent a week investigating the industry project in a selected area of career interest. The industries included commerce, engineering, information and communication technologies and research.

The industry partners were the Reserve Bank of Australia (RBA), Downer Group, Ford, IBM, the Australian Council for Educational Research (ACER), Commonwealth Scientific and Industrial Research Organisation (CSIRO) and Gene Technology Access Centre (GTAC).

Each of these industries provided mentors who worked with the students. The mentors, either mathematicians or professionals (e.g. research scientists, engineers) in fields utilising mathematics, provided the industry project and then supported the student team throughout the week (both face to face and online) in solving the problem.

Whilst each group of students worked on the project from their chosen career of choice, they also had the opportunity to experience a site visit to industries from each of the other three fields, and the DST (Defence, Science and Technology Group), where they heard mathematicians or industry mentors share their experiences inspiring students to similar occupations.

Each afternoon the students went to RMIT and worked on their projects supported by mathematicians (including lecturers and PhD students) from Melbourne's universities and research institutes. The camp culiminated with each group giving a presentation at LaTrobe University Bundoora, attended by family members, industry and university mentors. During the presentations students outlined their project, processes undertaken and conclusions reached.

Students experiencing the virtual reality lab at Ford.

STUDENT REFLECTIONS

Zavier Evans
South Gippsland Secondary College
Research, CSIRO

My group designed the Benalla water system, factoring in the demand of the people, how much the two dams in Benalla could hold, evaporation rate of water, rainfall and then find the maximum yield level (the point at which the demand of the people would outrun the water system's capacity). If you are in Year 9 and love mathematics, then remember to apply - it isn't an opportunity to pass up.

Lauren Gingell
Ballarat Clarendon College
Scientific Research, GTAC

We investigated which gene in mitochondrial DNA is the most appropriate to show evolutionary traits. I saw first-hand how mathematical principles can aid advancements in biological sciences and the mentors taught us the basic principles of genetics. In one laboratory, we explored the biological theories and methods around how genetic sequences are identified, then applied this knowledge to establish and to read a short section of genetic code.

The camp showed me that there are many opportunities in mathematics and science. More than ever, I want to be part of it.

My view on mathematics and its application in everyday life has changed quite considerably. Prior to the camp, I had almost no idea of how learning maths was such an integral part of so many fields and the opportunities that studying maths gives you when investigating potential career paths.

Zara Coulston-Williams
Tallangatta Secondary College
Research, ACER

We investigated the relationship between student ability and question difficulty for dichotomous items using the exponential formula that we were given. I enjoyed

Gene sequencing at GTAC.

RBA economist explains the relationship between unemployment and inflation.

At IBM students witnessed the use of mathematics in robotic artificial intelligence.

the camp and made a lot of a friends and learning quite a bit about how mathematics can be applied to the modern world.

Tully Sonneman
Sacred Heart College, Kyneton
Reserve Bank Australia

I learnt that the banks have a much bigger job in the economy than I thought. They set the cash flow which controls the unemployment and inflation rates. I now know that the structure of the economy is in the hands of the banks.

The maths camp confirmed that I want to do something mathematical in the future.I had a great time and learned about the real life industries that use mathematics daily.

THANK YOU

The MAV acknowledges the support of the industry mentors who provided projects, mentored students and offered a site visit.

- Reserve Bank of Australia
- Downer Group
- Ford
- IBM
- The Australian Council for Educational Research
- Commonwealth Scientific and Industrial Research Organisation
- Gene Technology Access Centre

Thanks also to the Defence, Science and Technology Group for a site visit, AMSI and in particular the university lecturers and PhD students who supported the students and the mathematics faculties of RMIT and La Trobe University for making their premises available. The MAV thanks the teachers who attended the camp and assisted in supervision and mentoring.

The MAV acknowledges the Department of Education and Training Victoria for their support in this project.

2017 MATHS CAMP

In Term 4, the MAV will be seeking expressions of interest from 20 regional Year 9 students to participate in the 2017 program. For information contact Helen Haralambous, hharalambous@mav.vic.edu.au or Julie Allen, jallen@mav.vic.edu.au.

THE BISECTION METHOD AND NEWTON'S METHOD

Brian Stokes - Teaching Associate, Department of Mathematical Sciences, Monash University

In the current VCE Mathematics Study Design (2016 – 2018) for the subject Mathematical Methods, additional algebra material has been added as follows:

- Numerical approximation of roots of simple polynomial functions using the bisection method and
- Numerical approximation of roots of cubic polynomial functions using Newton's method.

The purpose of this article is to discuss these two methods and to illustrate their application with examples. Exercises are provided together with answers.

THE BISECTION METHOD

The bisection method is a root-finding method that repeatedly bisects an interval and then selects a sub-interval in which a root must lie.

Consider the function $y = f(x)$ which is continuous on the closed interval $[a, b]$. If $f(a)\,f(b) < 0$, the function changes sign on the interval (a, b) and, therefore, has a root in the interval. The bisection method uses this idea in the following way.
If $f(a)\,f(b) < 0$, then we compute
$c = \frac{1}{2}(a + b)$ and test whether $f(a)\,f(c) < 0$.

If this is so, then $f(x)$ has a root in $[a, c]$. So c is now reassigned as b and we start again with the new interval $[a, b]$ which is now half as large as the original interval. If, on the other hand, $f(a)\,f(c) > 0$, then $f(c)\,f(b) < 0$ and c is now reassigned as a. In either case a new interval trapping the root has been found. The process can then be repeated until the required level of accuracy has been attained. Figures 1 and 2 illustrate the two cases discussed assuming $f(a) > 0$ and $f(b) < 0$. The bisection method is sometimes referred to as the method of interval halving.

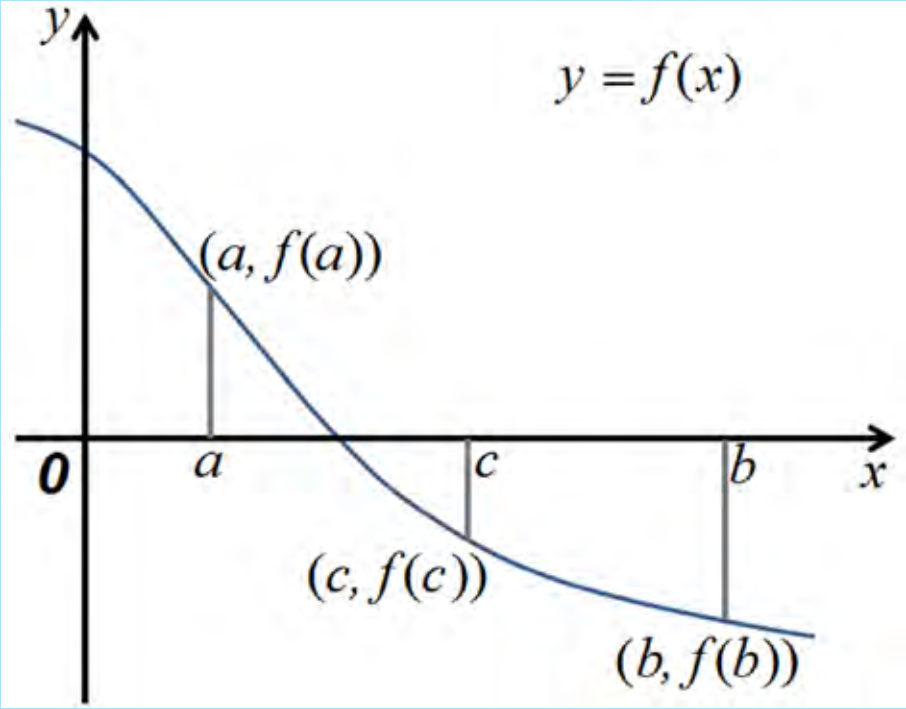


Figure 1. The bisection method selects the left subinterval. The root is to the left of $x = c$ and $f(a)\,f(c) < 0$.

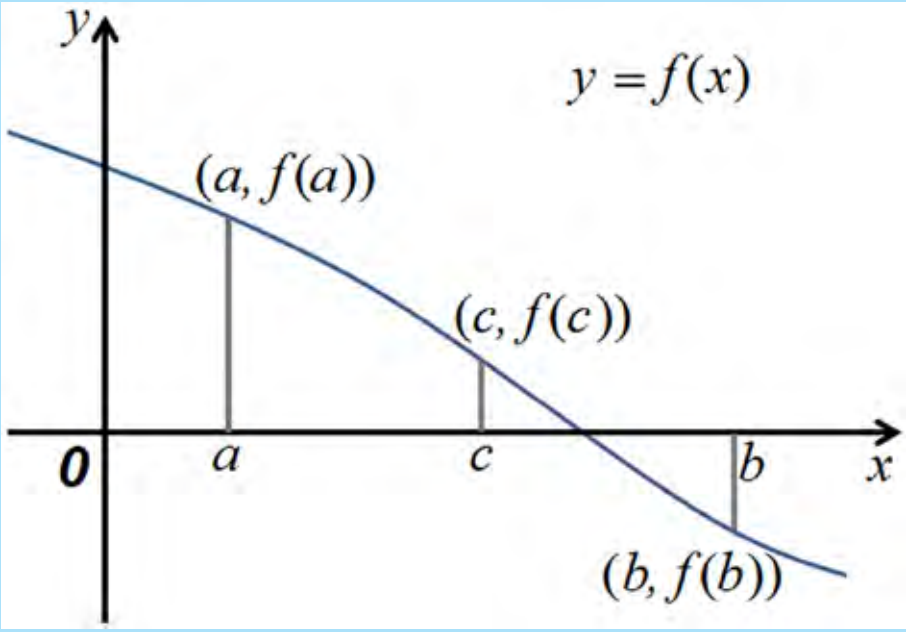


Figure 2. The bisection method selects the right subinterval. The root is to the right of $x = c$ and $f(c)\,f(b) < 0$.

EXAMPLE 1

Consider the continuous function $f(x) = x^3 + x - 1$.

(a) Evaluate $f(0)$.

(b) Evaluate $f(1)$.

(c) Determine the sign of $f(0)\,f(1)$.

(d) What conclusion can you draw from (c)?

(e) Use the bisection method to obtain the root of $y = f(x)$ to four decimal places.

SOLUTION TO EXAMPLE 1

(a) $f(0) = -1$

(b) $f(1) = 1$

(c) $f(0)\,f(1) = (-1) \times (1) = -1 < 0$, i.e., the sign of $f(0)\,f(1)$ is negative.

(d) $0 < x_{\text{root}} < 1$

(e) See Table 1.

EXAMPLE 2

Consider the function $f(x) = x^3 + 10x^2 + 8x - 50$.

(a) Evaluate $f(1)$.

(b) Evaluate $f(2)$.

(c) Evaluate $f(1)f(2)$.

(d) What is the sign of $f(1)f(2)$?

(e) What conclusion can you draw concerning your answer to part (d)?

(f) Use the bisection method to obtain the positive root of $y = f(x)$ to four decimal places.

SOLUTION TO EXAMPLE 2

(a) $f(1) = -31$

(b) $f(2) = 14$

(c) $f(1)f(2) = -31 \times 14 = -434 < 0$

(d) The sign of $f(1)f(2)$ is negative.

(e) $1 < x_{root} < 2$

(f) See Table 2.

THE BISECTION METHOD EXERCISES

QUESTION 1

Use the bisection method to find the root of $f(x) = 5x - 9$ to one decimal place. Take $a = 1$ and $b = 3$.

[ANSWER: $x_{root} = 1.8$]

QUESTION 2

Using the bisection method, obtain the greater positive root of $f(x) = x^2 - 6x + 7$ to four decimal places.

[ANSWER: $x_{root} = 4.4142$]

QUESTION 3

Obtain the greatest positive root to four decimal places of $f(x) = -x^3 + 9x^2 - 20x + 6$ using the bisection method over the interval [5,6].

[ANSWER: $x_{root} = 5.6458$]

QUESTION 4

Use the bisection method to obtain the co-ordinates of the point of intersection, to an accuracy of four decimal places, of $y = -x^3 + 4x^2 - 3x + 2$ and $y = 2x - 7$ over the interval [2,4].

[ANSWER: $(x,y) = (2.8637, 2.7274)$]

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
0	1	0.5	-1	1	-0.375
0.5	1	0.75	-0.375	1	0.1719
0.5	0.75	0.625	-0.375	0.1719	-0.1309
0.625	0.75	0.6875	-0.1309	0.1719	0.0125
0.625	0.6875	0.6563	-0.1309	0.0125	-0.0610
0.6563	0.6875	0.6719	-0.0610	0.0125	-0.0248
0.6719	0.6875	0.6797	-0.0248	0.0125	-0.0063
0.6797	0.6875	0.6836	-0.0063	0.0125	0.0031
0.6797	0.6836	0.6817	-0.0063	0.0031	-0.0015
0.6817	0.6836	0.6827	-0.0015	0.0031	0.0009
0.6817	0.6827	0.6822	-0.0015	0.0009	-0.0003
0.6822	0.6827	0.6825	-0.0003	0.0009	0.0004
0.6822	0.6825	0.6824	-0.0003	0.0004	0.0002
0.6822	0.6824	0.6823	-0.0003	0.0002	-0.0001
0.6823	0.6824	0.68335	-0.0001	0.0002	0.0001
0.6823	0.68335	0.68233	-0.0001	0.0001	0.00001
0.6823	0.68233	0.68232	-0.0001	0.00001	-0.00002
0.68233	0.68232	0.68233	0.00001	-0.00002	0.00001
0.68232	0.68233	0.68233	-0.00002	0.00001	0.00001

Table 1. After 17 iterations, we can safely take x_{root} to be 0.6823 to four decimal places.

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
1	2	1.5	-31	14	-12.125
1.5	2	1.75	-12.125	14	-0.0156
1.75	2	1.875	-0.0156	14	6.7480
1.75	1.875	1.8125	-0.0156	6.7480	3.3059
1.75	1.8125	1.7813	-0.0156	3.3059	1.6328
1.75	1.7813	1.7657	-0.0156	1.6328	0.8075
1.75	1.7657	1.7579	-0.0156	0.8074	0.3976
1.75	1.7579	1.7540	-0.0156	0.3976	0.1934
1.75	1.7540	1.7520	-0.0156	0.1934	0.0888
1.75	1.7520	1.7510	-0.0156	0.0888	0.0366
1.75	1.7510	1.7505	-0.0156	0.0366	0.0105
1.75	1.7505	1.75025	-0.0156	0.0105	-0.0026
1.75025	1.7505	1.75038	-0.0026	0.0105	0.0042
1.75025	1.75038	1.75032	-0.0026	0.0042	0.0012
1.75025	1.75032	1.75029	-0.0026	0.0011	-0.0005
1.75029	1.75032	1.75031	-0.0005	0.0011	0.0006
1.75029	1.75031	1.75030	-0.0005	0.0006	0.00003
1.75029	1.75030	1.750295	-0.0005	0.0003	-0.0002
1.750295	1.750295	1.750295	-0.0002	-0.0002	-0.0002

Table 2. After 17 iterations we can confidently deduce that x_{root} is 1.7503 to four decimal places.

THE BISECTION METHOD AND NEWTON'S METHOD (CONT.)

QUESTION 5

Obtain, to an accuracy of four decimal places, the co-ordinates of the point of intersection of the cubic functions $y = 2(x-1)^3$ and $y = -3(x-2)^3$ using the bisection method over the interval [1,2].

[ANSWER: $(x,y) = (1.5337, 0.3041)$]

NEWTON'S METHOD

Newton's method, also referred to as the Newton-Raphson Iteration Technique, involves less iterations than the bisection method since its convergence is quadratic rather than linear.

The basic idea is that if x_0 is an approximation to the root, x_{root}, of the equation $f(x) = 0$, then a closer approximation will be given by x_1 where the tangent to the graph at $x = x_0$ cuts the x-axis at $x = x_1$ as shown in Figure 3.

Using the definition of derivative at $x = x_0$

$$f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

$$\therefore (x_0 - x_1)\ f'(x_0) = f(x_0)$$

$$\therefore (x_0 - x_1) = \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \text{Eqn 1}$$

More generally, we may write

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{Eqn 2 where } n = 0, 1, 2, 3....$$

This equation is known formally as the Newton-Raphson Iteration Procedure for obtaining an approximation to the root of $f(x) = 0$.

EXAMPLE 1

Use Newton's method to find the positive root of $f(x) = x^3 - 6x^2 - 9x - 1$ to an accuracy of four decimal places. Take the initial guess to be $x_0 = 6$.

SOLUTION TO EXAMPLE 1

$$f(x) = x^3 - 6x^2 - 9x - 1$$

$$f'(x) = 3x^2 - 12x - 9 = 3(x^2 - 4x - 3)$$

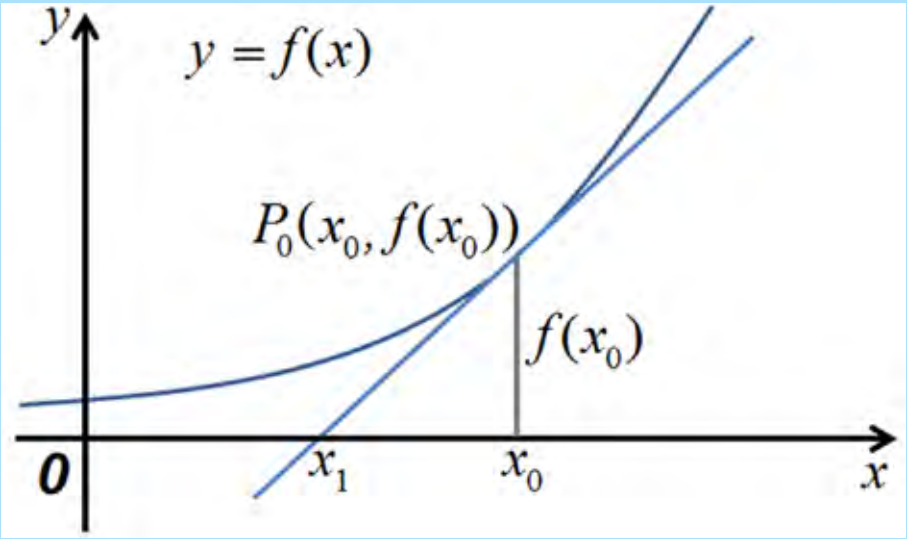


Figure 3. Newton's method for finding roots.

We shall take the initial guess to be $x_0 = 6$.

In accord with Equation (1), we define

$$k(x) = x - \frac{f(x)}{\frac{d}{dx} f(x)}$$

Initial guess is $x_0 = 6$.

We now carry out the following procedure using the Casio ClassPad II CAS calculator.

Define $f(x) = x^3 - 6x^2 - 9x - 1$

Define $k(x) = x - \dfrac{f(x)}{\frac{d}{dx} f(x)}$

6 EXE

$k(ans)$ EXE 8.0370

EXE 7.3777

EXE 7.2623

EXE 7.2588

EXE 7.2588

After four iterations we obtain the required root correct to four decimal places as follows: $x_{root} = 7.2588$

Clearly, Newton's method is more efficient and substantially faster than the bisection method.

EXAMPLE 2

Find the positive root of $f(x) = 4x^3 + 12x^2 - 32x - 29$ to four decimal places using Newton's method. Take the initial guess x_0 to be 1.5.

SOLUTION TO EXAMPLE 2

$$f(x) = 4x^3 + 12x^2 - 32x - 29$$

$$f'(x) = 12x^2 + 24x - 32 = 4(3x^2 + 6x - 8)$$

Initial guess is $x_0 = 1.5$

Again using the Casio ClassPad II CAS calculator, we carry out the following procedure:

Define $f(x) = 4x^3 + 12x^2 - 32x - 29$

Define $k(x) = x - \dfrac{f(x)}{\frac{d}{dx} f(x)}$

1.5 EXE

$k(ans)$ EXE 2.6774

EXE 2.2706

EXE 2.1872

EXE 2.1837

EXE 2.1837

After four iterations we obtain the positive root to this cubic function to four decimal places as follows: $x_{root} = 2.1837$.

EXERCISES – NEWTON'S METHOD

QUESTION 1

Use Newton's method to find the greatest root of $f(x) = x^3 - 4x^2 - 2x + 4$ to four decimal places. Take the initial guess to be $x_0 = 4.5$.

[ANSWER: $x_{root} = 4.2491$]

QUESTION 2

Find the root of $f(x) = 2x^3 - 4x^2 + 5x - 7$ to four decimal places using Newton's method. Take the initial guess to be $x_0 = 1$.

[ANSWER: $x_{root} = 1.7263$]

QUESTION 3

The function $f(x) = x^3 - 7x + 7$ has two roots on the interval $[1,2]$. Find these two roots to four decimal places.

[ANSWER: $x_{root1} = 1.3569, x_{root2} = 1.6920$]

QUESTION 4(a)

Find the square root of 17 to four decimal places using Newton's method.

Hint: Let $f(x) = x^2 - 17$. Then $f'(x) = 2x$. The iteration formula becomes

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{({x_0}^2 - 17)}{2x_0}$$

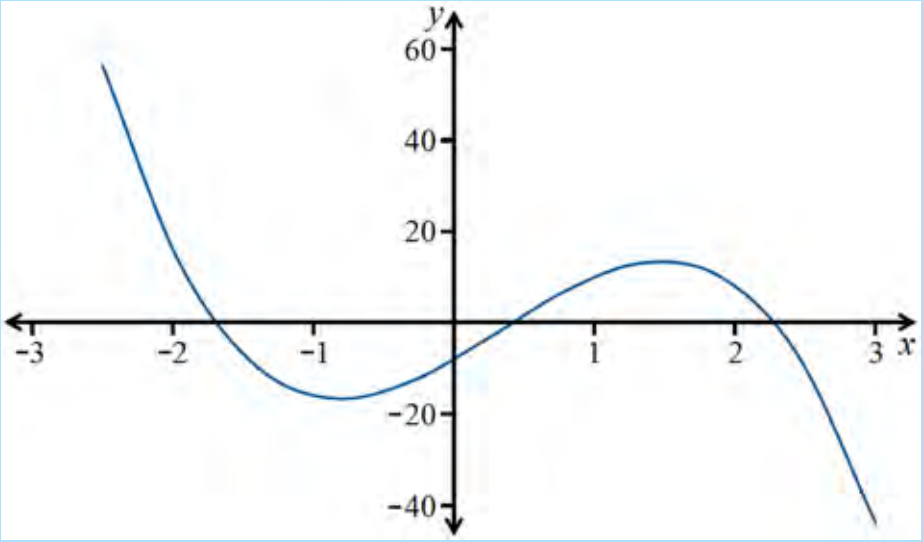


Figure 4: $f(x) = -5x^3 + 5x^2 + 18x - 8$

$$= x_0 - \frac{x_0}{2} + \frac{17}{2x_0}$$

$$= \frac{x_0}{2} + \frac{17}{2x_0}$$

$$= \frac{1}{2}\left(x_0 + \frac{17}{x_0}\right)$$

Take x_0 to be 4 and start the iteration process.

[ANSWER: 4.1231 after only two iterations]

QUESTION 4(b)

Find the cube root of 28 to four decimal places using Newton's method.

[ANSWER: 3.0366]

QUESTION 5

Use Newton's method to find all three roots to four decimal places of the function $f(x) = -5x^3 + 5x^2 + 18x - 8$. Take the initial guesses to be $-2, 0.5$ and 2. See Figure 4.

[ANSWER: $x_{root} = -1.6901, 0.4163$ and 2.2738]

MATHEMATICS AND SPORT

Ian Lowe - Mathematics Education Consultant, MAV

Faster, Higher, Stronger: The Olympic display inside the National Sports Museum includes artefacts dating back to 1896, and is highlighted by the cauldron from the 1956 Melbourne Games, and Cathy Freeman's gold-winning running suit from the 2000 Sydney Olympics.

PROJECT MATERIAL FOR YEARS 5 TO VCAL

MAV has provided materials relating mathematics and sport, and it is available free on the website of the National Sports Museum (NSM): www.nsm.org.au/School%20Excursions/Learning%20Resources/Mathematics.aspx, or go to www.nsm.org, choose School Excursions, Resources and then Mathematics.

There are four documents and they relate to a possible (but not compulsory) visit to the NSM situated at the MCG, a short walk from Jolimont railway station.

- **Before the visit:** *How is maths used in sports* - an introduction to the spots and the maths, and the concept of a project linking the two.
- **At the NSM:** *Mathematics at the National Sports Museum* - questions and answers to match while at the museum. There are primary and secondary versions.
- **After the visit back at school:** *Pick your sports* - an electronic resource (pdf) for use as a project back at the school or for homework. It includes many links to pdfs, spreadsheets and the internet. There are ten sports involved: athletics, AFL, basketball, cricket, cycling, golf, horse racing, soccer, swimming and tennis.
- **For the teacher:** *Teaching ideas for Mathematics and Sport* - to help you guide your students to mathematics suitable for their level, this document shows the Victoria Curriculum level for different sections of the *Pick your sports* material.

There are many ways to use these materials back at your school.

- You might let students explore the whole package and write an essay about how mathematics is used in sport. The essay should contain some examples they have found.
- You might ask students to choose a sport and explore it in depth. It is well known that students who are really interested in a context of real-life mathematics (such as a favourite sport) can far exceed the level at which they might normally perform. In this case we are using a theme to motivate learning. It works! However you will know that the students are at different points along their learning journey in mathematics, so they should not be asked to work other than in groups who are at a similar level.
- You might also try to select bits of one or more sports and use them to supplement your regular curriculum. It is possible that this will have less success, unless all students are interested in the same sport.

Check out the mathematics resources at www.nsm.org.au and let us know what you think by emailing Ian Lowe, ilowe@mav.vic.edu.au.

REVIEW: TEACHING WITH TENS FRAMES

TEACHING WITH TEN FRAMES
DR PAUL SWAN AND KELLY NORRIS

Teaching with Ten Frames provides exceptional and useful activities and games that develop fluency and mental computation of sequencing, subitising, addition and subtraction skills. With references to the Australian Curriculum, year levels, and blackline masters, this resource is almost too easy to use.

As a graduate teacher still finding my feet in the world of Prep, this is an exceedingly valuable resource for any beginning teacher or education professional looking to change-up or add to their current maths program.

With particular focus to any of the spinner games, I have witnessed the positive learning gained from student engagement and enjoyment with these activities.

A big plus of the book is the ability to provide multiple points of entry and allow for specific targeting students' needs.

From basic subitising to recognising 10 more or less than a given number, I have been able to support my students with varying levels of knowledge and achievement to provide challenging learning experiences.

I teach 35 Prep students, and these activities have provided excellent learning opportunities for my students and can be used as an introduction to number topics, warm-up activities, teacher focus groups, or rotational group activities.

Ridiculously simple, yet incredibly effective; providing students with basic skills necessary for years P-2 in a fun and organised fashion.

Jono Schmidt, Prep Teacher
Stonnington Primary School.

Teaching with Ten Frames is available from the MAVshop, order online at http://shop.mav.vic.edu.au. Member price $21.45.

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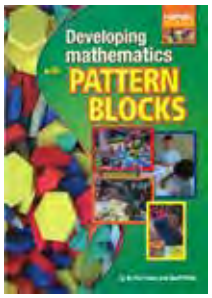

HANDS ON MATHS SERIES: DEVELOPING MATHEMATICS WITH PATTERN BLOCKS

2-6

A teacher reference resource, which also includes reproducible student worksheets. It provides a range of developmental activities to encourage the best use of pattern blocks in the classroom. The book includes a comprehensive background information including organisation, objectives, importance of language and detailed step-by-step instructions, clear illustrations and photographs for ease of use.

$42.09 (MEMBER)
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TEACH MATHS FOR UNDERSTANDING: RESOURCES PACKAGE FOR SECONDARY

F-12

This set contains the following 17 CDs: Certain Number, Dice don't have brains, Active Learning, Active Learning 2, Geometry everywhere, Guidelines in Number, Guidelines in Measurement, Interactive Learning, People Count, RIME, RIME 5&6, Careers and Maths, Maths at Work, Maths at Work 2, Hands-on Ratio, MApps, Understanding the Calculus with spreadsheets.

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OTHER TEACHING METHODS

6-8

Christine Boyer has produced a set of hands-on group learning resources for you to copy and use in groups in your classroom. Most of these resources use nine different cooperative games formats that involve matching concepts, meanings and symbolism for all sections of AusVELS/AC. Teachers will find that students help one another while strengthening their understanding of key mathematical ideas. The set is accompanied by a delightful fantasy story to help learners understand fractions. They are available on CD or in book form.

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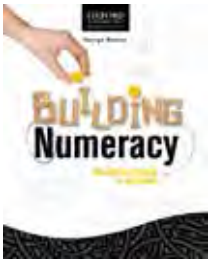

BUILDING NUMERACY

3-10

Building numeracy in all students is a critical aspect of contemporary schooling. *Building Numeracy* has been written to assist all teachers, at all levels, to develop expertise and confidence in diagnosing student difficulties. This book helps teachers to understand how mathematical concepts and processes are constructed and connected; overcome mathematical misconceptions and inappropriate ways of thinking; plan and implement appropriate intervention programs to build students' competence and confidence. A series of numeracy screening tests are provided to allow teachers to readily assess a class, small group or individuals. The tests provide a picture of strengths and weaknesses or gaps in understanding. Included is a CD-ROM containing a variety of digital resources designed to support the diagnosis and intervention processes. It also includes a series of instructional videos that demonstrate the effective teaching of key concepts and strategies.

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