THE MAGIC OF 1-9

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The essence of a successful mathematics program lies in both its relevance and its ability to engage students. The activities found in this article are highly engaging and enable students to exercise their abilities to apply their acquired skills and concepts to unfamiliar situations, the true nature of problem solving.

The digits from 1 to 9 can be manipulated in a number of ways to produce solutions that appear to students to be almost 'magical'. Many of the activities found in this article have numerous possible answers ,that add to the mystique of the nature of our number system and, as a consequence, both challenge and delight. They encourage co-operative group work, (learning is essentially a social activity), and at the same time afford a 'low threshold and high ceiling' approach to differentiation.

ACTIVITY 1
THE 1- 9 MAGIC SQUARE

An excellent starting point is the well known magic square where students are asked to enter the digits from 1 to 9 into a 3 x 3 square in such a manner that the three rows, three columns and two diagonals all sum to the same total.

4	9	2
3	5	7
8	1	6

Why is five in the centre of the magic square? Are different solutions possible?

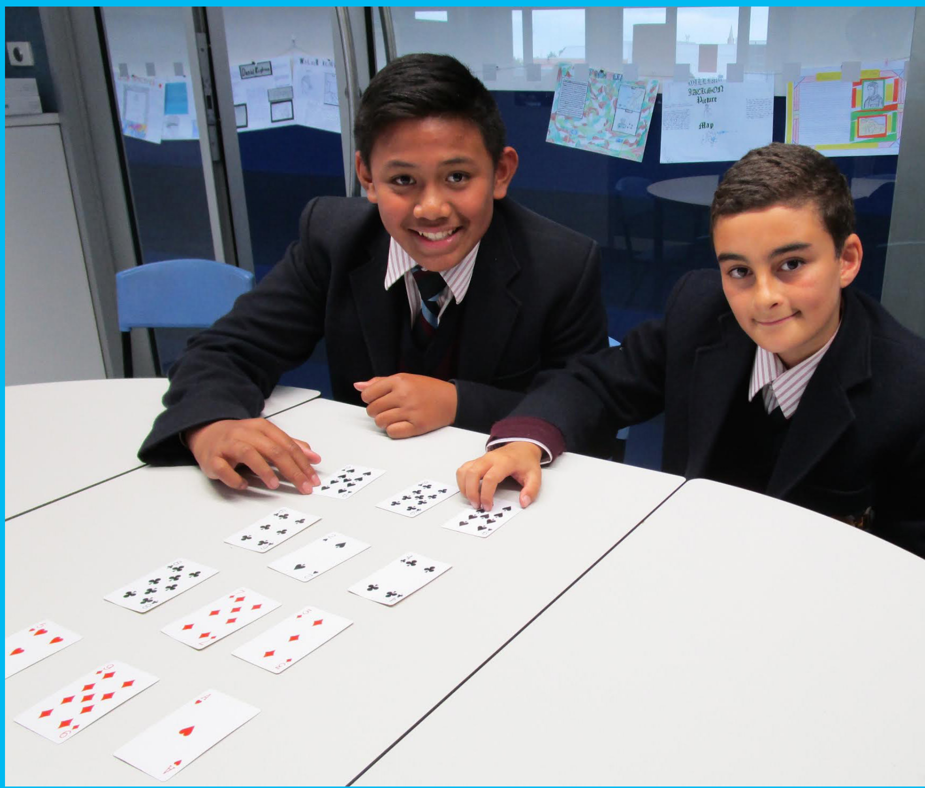
Can answers be found that do not have five in the centre?

ACTIVITY 2
MAKE 999

This exercise asks the students to use the digits from 1 to 9 to create three three-digit numbers that sum to 999. One possible solution is:

195
378
<u>426</u>
999

One of the most intriguing things about this task is the number of different solutions that work. Encourage students to check their answers with a calculator.



Make 999

Bear in mind, that in this instance, the calculator is not being used as the computational tool; its implementation is legitimised by being used to verify the students' work.

List the different solutions to enable the students to find patterns in the configurations that work, (where must the eight and the nine be found?). Remember that mathematics is basically the study of patterns and connections.

ACTIVITY 3
1-9 CARD CROSS

Ask the students to arrange number cards (or playing cards) into a cross in such a way that the two arms of the cross sum to the same value. One possible result is:

		9		
		7		
8	2	3	5	6
		1		
		4		

Are other results possible?

Must three be the pivotal point for the cross? Will the arms always sum to 24?

ACTIVITY 4
THE EINSTEIN PROBLEM

It has been suggested that at the age of six Albert Einstein was asked to arrange the digits from 1 to 9, in that order, to create an equation that equalled 100. Supposedly he wrote down
$1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100$.

Apocryphal or not, what has become known as the Einstein Problem is a wonderful way to exercise students' estimation skills and operational competence. What is rather amazing about this question is the fact that there are at least 10 other solutions that work. Perhaps the most elegant is $123 - 45 - 67 + 89$, requiring the use of only three operational signs. Once again, the students should be encouraged to check their estimates by using the calculator. However, a trap for young players is the use of the calculator to find a solution of 100 only to realise that the equation had not been written down first. Tantalisingly, there are numerous solutions equalling 99 and 101!

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Due to the challenging nature of this task it is a worthwhile tip to give the less able mathematicians in the class a decent start, such as the first one or two steps contained within the equation.

Further extension and further very useful applications of the problem lie in the realm of decimals. Allow rational numbers to be incorporated within the equations and answers such as:

1.2 + 0.3 - 4 + 0.5 + 6 + 7 + 89

become possible. Again, as is the case with whole numbers, numerous solutions are possible when, if one pardons the dreadful pun, decimals are brought into the equation.

ACTIVITY 5
1 – 9 EQUALITY

This activity encourages students to consider fractions as ratios. Hopefully the days are now long gone when teachers advice students that to form an equal fraction 'you do to the top what you do to the bottom'. This classic rote oriented way of instruction may well result in the students getting a tick but will never result in a student developing a deep understanding of the nature of equal fractions. Yes, to form an equal fraction both the numerator and the denominator need to be multiplied by the same number, but these numbers must be seen as forming a fractional name for one whole number. Because the law of identity tells us that any number multiplied by one must in essence, remain the same, pairs of equal fractions fulfil the effects of this law.

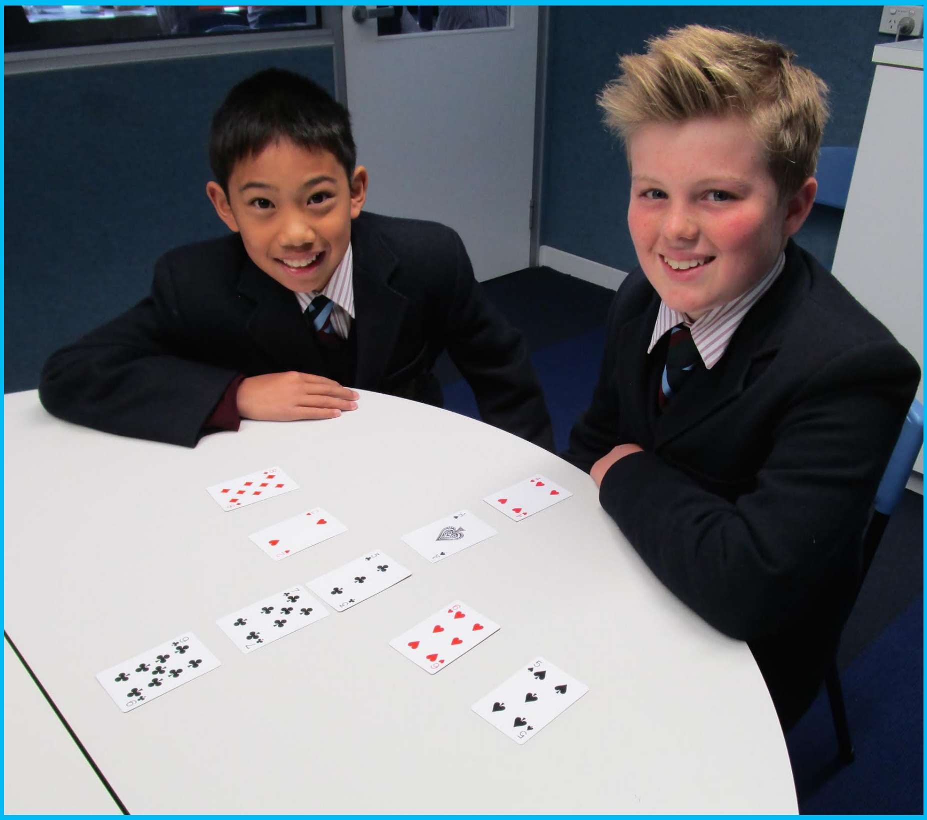
Equal fractions can also be seen as demonstrating equal ratios.

$\frac{16}{32}$ and $\frac{4}{8}$ must be equal fractions

because the ratio of the numerators to the denominators is 1:2 or conversely 2:1 when the denominators are compared to the numerators.

This activity requires using the numbers from 1 to 9 to create a name for $\frac{1}{2}$.

Logic tells us that the fraction must contain four of the digits in the numerator and five in the denominator and that the denominator must start with one.



Card Cross

Common sense would seem to suggest that it would be remarkable to find an answer that works, like:

$$\frac{9273}{18546}$$

How truly amazing it is to be able to find multiple answers that have numerators starting with six, seven and nine. Pile on the wonder as your students come to the realisation that the structure of this question will lead to multiple fractional answers equalling

$\frac{1}{3}$ and $\frac{1}{5}$.

My students and I could only find one that equalled $\frac{1}{4}$ but there may be more.

CONCLUSION

Mathematics is a subject that contains a deal of inherent beauty. The activities outlined in this article will hopefully demonstrate this point to your students and at the same time inspire and challenge them to discover magical solutions.

REFERENCES

All You Need To Teach Calculators, Macmillan Education 2010

Macmillan Problem Solving Boxes 1 – 6, Macmillan Education, 2011

Maths Games On The Go, Macmillan Education, 2007

Teach Maths For Understanding – The Mathematical Association of Victoria

Penleigh and Essendon Grammar School is an MAV accredited Mathematics Active School.

To share your innovative card games or for more ideas about how to use card and dice games to enhance your teaching and learning program contact MAV's mathematics education consultants on 9380 2399.

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