Privileging understanding and prioritising proficiency - Addressing the STEM challenge

Keynote presentation to the Annual Conference of the Mathematical Association of Victoria, 3 December 2015

by Professor Dianne Siemon
RMIT University
Overview:

- The STEM imperative
- So, what’s the solution?
- Lessons from research
- Multiplicative thinking – a really BIG IDEA
- A STEM initiative – focussing on multiplicative thinking and mathematical reasoning
- Targetted teaching works - an example from NSW

It took 3 men 3 days to paint the inside of the house. How long will it take 2 men?
The STEM imperative

44% or 5.1 million jobs at risk from digital disruption

Shifting 1% of workforce into STEM roles would add $57.4 billion to GDP over 20 years

75% of the fastest growing occupations require STEM

The average performance of Year 8 students in mathematics has not changed since TIMSS 1995

More than 20% of Year 8 students were being taught by mathematics by teachers who reported feeling only “somewhat” confident in teaching the subject

37% of Australian Year 8 students did not achieve the Intermediate international benchmark (the minimum proficient standard expected)

Number of Year 12 students studying STEM subjects is declining

1. Price-Waterhouse Report (April, 2015). A Smart Move: Future proofing Australia’s workforce by growing skills in science, mathematics, engineering and maths (STEM)
The number of students taking intermediate and advanced maths at secondary school has fallen by 34% over the last 18 years.

Australia’s mean mathematical literacy performance declined significantly between PISA 2003 and PISA 2012 and males significantly outperformed females.

44% of employers experience difficulties recruiting STEM qualified technicians/trade workers … lack qualifications relevant to the business (36%) … lack employability skills and workplace experience (34%).

Interpreting, applying and evaluating mathematical outcomes an area of relative strength for Australian 15-year olds but formulating situations mathematically and employing mathematical concepts, facts, procedures and reasoning are areas of weakness.

20% of mathematics and physics teachers are teaching out-of-field.

Mathematics is central to STEM

It is the only subject whose study consistently enhances performance across all fields of science. It forms the basis of most scientific and industrial research and development. As an enabling discipline, mathematics, has been, and will continue to be at the heart of our search for ways to solve, manage, mitigate or adapt to some of the great challenges that confront us as a nation ans as part of humankind. If we are to face these challenges we must ensure a steady supply of mathematically trained graduates and a workforce and a community with the mathematical skills required.

The Chief Scientist, Professor Ian Chubb, September 2014
So, what’s the solution?

My instincts tell me that a back-to-basics approach to education is what the country is looking for, what parents feel comfortable about.

Minister for Education, Christopher Pyne

23 September 2013
Demonstrate and practice?

What’s the point?

What do you think of questions 1, 7, 9 and 12?
Fluency without understanding

At what cost?

Big Ideas:
- Place Value
- Multiplicative Thinking
A Year 8 student’s response to the question: What sort of maths do you really like doing?

Note the reliance on a count of groups, that is, the equal groups idea for multiplication and the corresponding quotation idea for division

This is perpetuated in interpretations such as $3x = x + x + x$ and $2 \div \frac{1}{2}$ as ‘how many halves in 2’

Big Ideas:
- Trusting the Count
- Place Value
- Multiplicative Thinking
So, what’s the solution?

Demonstrate and practice – modify the learner

Mastery learning (Bloom) – modify time, individualise materials

Direct Instruction (Engelmann, Huitt) – modify the curriculum and pace of learning (adaptive learning?)

Differentiated instruction (Tomlinson) – modify learning style, student interest

Adaptive learning – modify tasks according to student responses in an interactive environment

Streaming ??

Textbooks ??
Lessons from research

*Middle Years Numeracy Research Project (MYNRP)* large scale, cross-sectoral project in Victoria (1999-2000) – explored number sense, measurement & data sense and spatial sense using rich tasks and Item Response Modelling (IRM) - identified *multiplicative thinking* as the area most responsible for the 7 year range in student mathematics achievement in Years 5 to 9 (Siemon, Corneille & Virgona, 2001)*

*Scaffolding Numeracy in the Middle Years (SNMY)* Linkage Project with the Departments of Education in Victoria and Tasmania (2003-2006) - explored the development of *multiplicative thinking in Years 4 to 8* using rich assessment tasks and IRM– confirmed MYNRP result, 8 year range, produced research-based *Learning and Assessment Framework for Multiplicative Thinking (LAF)*, two formative test options and teaching resources for each zone in the LAF (Siemon, Breed, Dole, Izard & Virgona, 2006)*

* The Final Reports of both projects can be found on the DET website
The *Middle Years Numeracy Research Project* (MYNRP, 1999-2001)

- As much difference within Year levels as between Year levels (spread).
- Considerable within school variation (individual teachers make a significant difference to student learning);
- The needs of many students, but particularly those ‘at risk’ or ‘left behind’, were not being met.
- Differences in performance were largely due to an inadequate understanding of fractions, decimals, and proportion, and a reluctance/inability to explain/justify solutions.

Mean Adjusted Logit Scores by Location November 1999 (N=6879)

Recognised as multiplicative thinking (Vergnaud, 1983)

© Dianne Siemon
Notion of **targeted teaching** that requires:

- **access to assessment techniques** that expose student’s thinking;

- a **grounded knowledge of learning trajectories** (key steps in the development of big ideas and how to scaffold these);

- an **expanded repertoire of teaching approaches** which accommodate and nurture discourse, help uncover and explore student’s ideas in constructive ways, and ensure all students can participate in and contribute to the enterprise;

- **sufficient time with students to develop trust and supportive relationships**; and

- **flexibility to spend time with the students who need it most.**

(MYNRP, Final Report, 2001)
This is particularly important in relation to a relatively small number of ‘big’ ideas and strategies in Number, without which students’ progress in mathematics will be seriously impacted

(Siemon, 2006).
# The big ideas in Number F-10

<table>
<thead>
<tr>
<th>Year</th>
<th>Idea</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (Prep)</td>
<td><strong>Trusting the count</strong></td>
<td>Developing flexible mental objects for the numbers 0 to 10, part-part-whole knowledge</td>
</tr>
<tr>
<td>2</td>
<td><strong>Place-value</strong></td>
<td>The importance of moving beyond counting by ones, the structure of the base ten numeration system</td>
</tr>
<tr>
<td>4</td>
<td><strong>Multiplicative thinking</strong> (initial ideas)**</td>
<td>The key to understanding rational number and developing efficient mental and written computation strategies in later years</td>
</tr>
<tr>
<td>6</td>
<td><strong>Partitioning</strong> (equal parts)**</td>
<td>The missing link in building common fraction and decimal knowledge and confidence</td>
</tr>
<tr>
<td>8</td>
<td><strong>Proportional reasoning</strong></td>
<td>Extending what is known beyond rule-based procedures to represent and solve problems involving fractions, decimals, percent, ratio, rate and proportion</td>
</tr>
<tr>
<td>10</td>
<td><strong>Generalising/Formalising</strong></td>
<td>Skills and strategies to support equivalence, recognition of number properties and patterns, and the use of algebraic text</td>
</tr>
</tbody>
</table>

(Siemon, 2006; 2011)
Multiplicative Thinking – a very BIG IDEA

- a capacity to work flexibly and efficiently with an extended range of numbers including fractions, decimals and percents;

- an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion, rate and ratio; and

- the means to communicate this effectively in a variety of ways (e.g., words, diagrams, symbolic expressions, and written algorithms).

A muffin recipe requires 2/3 of a cup of milk. Each recipe makes 12 muffins. How many muffins can be made using 6 cups of milk?
Three solutions:

A muffin recipe requires 2/3 of a cup of milk. Each recipe makes 12 muffins. How many muffins can be made using 6 cups of milk?

Solutions which rely on counting all groups are essentially additive.

Solutions which rely on some form of proportional reasoning are essentially multiplicatively.
Multiplicative Thinking

The essential difference between additive and multiplicative thinking relates to the nature of the units under consideration.

In additive situations it is possible to work with the numbers involved as collections that can be aggregated or disaggregated and renamed as needed to facilitate computation.

In multiplicative situations, it is necessary to simultaneously recognise and coordinate the number of groups (multiplier) and the number in each group (multiplicand) to arrive at a composite of composite units (e.g. Ba-Ba Black Sheep problem, 15 bales of wool)

(Siemon, Beswick, Brady, Clark, Faragher & Warren, 2011).

The phone bill for May was $356. The bill for June was $504. How much more was paid in June?

Roast lamb takes 45 minutes/kilogram and 20 minutes extra. How long will it take to roast a 1.8 kilogram leg of lamb?
Young children’s solutions to the Baa-Baa Black Sheep problem: 5 sheep, how many bales of wool?
Scaffolding Numeracy in the Middle Years (SNMY Project, 2004-2006)

- **Multiplicative thinking** operationalised in terms of
  1. core content knowledge (multiplication, division, fractions, decimals, proportion etc),
  2. ability to apply that knowledge in unfamiliar situations, and
  3. capacity to communicate and justify solution strategies

- Hypothetical Learning Trajectory (Simon, 1995) for multiplicative thinking derived from related literature

- HLT used to locate, design, and trial rich assessment tasks

- Cluster-based purposeful sample of 3200 Year 4 to 8 students in Victoria and Tasmania, pre/post test design, support for targeted teaching

- Rasch analysis (e.g., Bond & Fox, 2001) used to identify shift over time and test HLT
SNMY Extended Task

BUTTERFLY HOUSE...

Some children visited the Butterfly House at the Zoo.

They learnt that a butterfly is made up of 4 wings, one body and two feelers.
While they were there, they made models and answered some questions.
For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

_________ wings
___________ bodies
___________ feelers

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

Adapted from ‘Butterflies and Caterpillars’ (Kenney, Lindquist & Heffernan, 2002) for the SNMY Project (2003-2006)

This task had 9 items altogether including:

Items of increasing complexity, eg, “How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers?”

Items involving simple proportion and rate, eg, “To feed 2 butterflies, the zoo needs 5 drops of nectar per day. How many drops would be needed per day to feed 12 butterflies?” …and

Items involving the Cartesian product, eg, given 3 different body colours, 2 types of feelers and 3 different wing colours, “How many different model butterflies could be made?”
Reading and interpreting quantitative data relative to context

Recognising relevance of proportion

Mathematics used, eg, percent, fractions, ratio

Open-ended question

Problem solving, solution strategy unclear

SNMY Short Task

ADVENTURE CAMP ...

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again. The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

<table>
<thead>
<tr>
<th></th>
<th>Rock Wall</th>
<th>Canoeing</th>
<th>Archery</th>
<th>Ropes Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 5</td>
<td>15</td>
<td>18</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>Year 7</td>
<td>19</td>
<td>21</td>
<td>38</td>
<td>22</td>
</tr>
</tbody>
</table>

Camp Reefton Thursday Activities

a. What can you say about the choices of Year 5 and Year 7 students?

b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director’s statement? Use as much mathematics as you can to support your answer.

SNMY Project (2003-2006)
ADVENTURE CAMP ...

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again.

The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Year 5</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock Wall</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>Canoeing</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>Archery</td>
<td>24</td>
<td>38</td>
</tr>
<tr>
<td>Ropes Course</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

Camp Reefton Thursday Activities

a. What can you say about the choices of Year 5 and Year 7 students?

The majority of both groups chose Archery while the other activities have around the same numbers except for the rock wall which more year 7 chose.

b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? Use as much mathematics as you can to support your answer.

The director is probably right because all together there are more year 7 than 5 so that the percentage of 5 would be higher than 7.

\[
\begin{align*}
&= \frac{18}{75} \times 100 = 24\% \\
&= \frac{6}{25} \times 100 = 24\%
\end{align*}
\]

A Year 6 Student Response to Adventure Camp Short Task (SNMY, May 2004)
Results ... Zone 4 can be viewed as a transitional zone from additive to multiplicative thinking, suggesting that about 40% of Year 7 and 30% of Year 8 students might be deemed to be ‘left behind’ in terms of curriculum expectations ...
A 7-8 year range in any one class ...

<table>
<thead>
<tr>
<th>LAF Zone</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected by</td>
<td>End of Year 1</td>
<td>End of Year 2</td>
<td>End of Year 3</td>
<td>End of Year 4</td>
<td>End of Year 5</td>
<td>End of Year 6</td>
<td>End of Year 7</td>
<td>End of Year 8</td>
</tr>
<tr>
<td>Year 4</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Year 5</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Year 6</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Year 7</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Year 8</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Implied class distribution by Year Level based on Initial 2004 SNMY data (N = 3169)
Targeted teaching works

For example, students in an identified sub-sample of ‘at-risk’ students within the SNMY Project demonstrated major shifts in achievement against the *Learning and Assessment Framework for Multiplicative Thinking* (LAF) as a result of an 18 week, 2 sessions per week teaching program* (Breed, 2011)

Participants: 9 Year 6 students identified at Level 1 of the Framework in May 2004

Results: All 9 students achieved at Level 4 or 5 of the Framework in November 2005

*A copy of the Intervention Teaching Program for At Risk Students is included in the SNMY Project Findings, Materials and Resources available on the DET and TasEd websites.*
Products of SNMY Research:

• A set of **valid and reliable tasks** that can be used with confidence to assess multiplicative thinking across Year levels;

• An evidence-based **Learning Assessment Framework for Multiplicative Thinking** that can be used to inform targeted teaching approaches;

• 8 **Learning Plans** per cluster (24 in all), one for each zone/level of the framework; and

• A number of school-based **authentic tasks**.

A STEM Initiative - AMSPP

The objectives of the *Australian Mathematics and Science Partnership Program* (2013-2017) are to:

(i) build the theoretical and pedagogical skills of school teachers to deliver maths and science subjects;

(ii) increase the number of school students undertaking maths and science subjects to Year 12;

(iii) improve outcomes for these students; and

(iv) encourage more students to study science, technology, engineering and maths (STEM) courses at university through innovative partnerships between universities, schools, and other relevant organisations.

Two AMSPP Project Rounds:

1. Priority Project Round (2013)

Submissions for one-year, ‘road-ready’ projects: total funding pool $5M, announced January 2013, awarded August 2013

*Reframing Mathematical Futures Priority Project* - focus on multiplicative thinking in Years 7 to 10 using the *Scaffolding Numeracy in the Middle Years* (SNMY) resources in 28 schools nationally


Submissions for extended research projects: total funding pool $19M, announced May 2013, awarded July 2014

*Reframing Mathematical Futures II Competitive Grant Project* – aimed at building a learning and teaching framework for algebraic, spatial and statistical reasoning in Years 5 to 9 – 32 schools nationally including NSW and WA
Reframing Mathematical Futures (RMF) (AMSPP Priority Project, 2013):

**Aim:** To improve multiplicative thinking and proportional reasoning in Years 7 to 10 using the SNMY materials and a supported targeted teaching approach …

Something more needed …

- At-risk student responses to MYNRP interviews (Siemon, Virgona & Corneille, 2001)
- Variable success of targeted teaching in secondary schools (SNMY Final Report, 2006)
- Role of affect and relationships in effective targeted teaching (Breed, 2011)
The views of students:

[When did you last enjoy maths?] “In class recently, doing fractions, changing fractions to decimals, it was good because I actually understood it and I felt better” (Matt, Year 6)

**Success** is crucial to engagement.

Relevance is about connectedness, not necessarily about immediately applicable, ‘real-world’ tasks, but about being able to access what is seen to translate to **further opportunities** to study ‘real maths’ and access to ‘good’ jobs.

**Self-esteem** - students believe that mathematics is important and relevant, they generally want to learn and be able to apply mathematics. Mathematics is not perceived to be as ‘boring’ or irrelevant as is often assumed.

(MYNRP, Final Report, 2001)
Views of ‘at risk’ students

“Change the way it’s explained, they need to think about how you understand, not how they explain” (Vincent, Year 9, MYNRP, 2001)

Disengagement has as much to do with student perceptions of how they are treated by their teachers as the teaching practices used …

A sense of cultural connectedness and mutual respect appears more likely to encourage constructive, risk-taking, explorative behaviour than feelings of alienation or uncertainty.

Engagement comes with self-esteem, identity and agency. It is a consequence of success not a pre-requisite for success.

It requires sufficient time with students to develop trust and supportive relationships and the flexibility to spend time with those who need it the most.

Adolescent Learners

Learn best when they:

• have high levels of confidence and self-esteem,
• are strongly motivated to learn, and
• are able to learn in an environment characterised by ‘high challenge coupled with low threat’.


Sagor and Cox (2004) identified five essential feelings they believe are crucial to a young person’s well-being and success at school:

• the need to feel **competent**,  
• the need to feel they **belong**,  
• the need to feel **useful**,  
• the need to feel **potent**, and  
• the need to feel **optimistic**.

Expectations of RMF Specialists:

• Identify participating teachers (two per school)
• Administer SNMY Assessment Options (at least four classes per school)
• Use project grants* to meet with team:
  o to mark and moderate SNMY responses
  o plan targeted teaching approach (when, where, how… CBUPO)
  o identify and source relevant resources and activities
  o review progress, share activities, build up resources
• Liaise with the project mentor to identify professional learning needs, seek advice
• Contribute to Collaborate sessions to share observations, resources, ideas and activities

* Two grants of $4500/school to fund time release, resources etc
Data Collection:

Data collected July/August and November 2013

- Complete SNMY data sets from just over 1700 students across Years 7 to 10 from 28 schools
- Student surveys (attitudes, perceptions of competence, belonging, usefulness, potency and optimism)
- Specialist and Teacher surveys (experience, pedagogical content knowledge, reflections)
- Field notes from school visits
- Artifacts (resources, photos, posters, planning documents)
- Student journals (where available)
- Principal report on funding, in-kind support, perceived value of project and future intentions
AMSPP Priority Project: SNMY Data

Complete (i.e., matched) data sets were obtained from 1732 students, with the majority of students in Year 8

<table>
<thead>
<tr>
<th>Year 7</th>
<th>Year 8</th>
<th>Year 9</th>
<th>Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>19%</td>
<td>59%</td>
<td>20%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Approximate proportion of students by Year Level

Student raw scores were translated to LAF zones/levels using the SNMY Raw Score Translator. Matched pairs were used to calculate effect size using the means and standard deviations of the pre and post test LAF zone/level data for each school. Effect sizes were extrapolated to one year and adjusted for regression to benchmark the results.

Result: an overall adjusted effect size of 0.65, which is well above Hattie’s (2012) benchmark effect size of 0.4, but considerable variation across schools
Overall improvement in student LAF Levels

47% in Zones 1-3 (Aug)
42% in Zones 1-3 (Nov)
22% in Zone 4 (Aug)
19% in Zone 4 (Nov)
31% in Zones 5-8 (Aug)
40% in Zones 5-8 (Nov)

Percentage of students at each LAF level, all students, August and November 2013 (N = 1732)
Understanding why…

Reasons for the differential results across schools, were explored at Project Workshop 2 in February 2014.

While there are a range of factors that impact student achievement, many of which are beyond the control of teachers and students, the following factors were offered as reasons for the significant improvement in student SNMY results.

• Teacher engagement with the process, use of LAF to adopt a more targeted teaching approach and share teaching ideas and resources
• Good use made of funds to support collaborative planning and resource development
• Availability of dedicated teaching spaces where differentiated resources could be stored and accessed easily
• Capacity to combine two or more classes to facilitate grouping by LAF Zones
• Team teaching and access to Specialist support staff
• Use of group work, concrete materials, student-directed activities, reflective journals and peer tutoring.
One of the recognised factors impacting student achievement is the **level of student engagement in the testing process**. As a result, in the November testing, schools were asked to rate each student by level of engagement using a scale of 1 (low) to 3 (high). While not all schools provided this data – the results are interesting for those that did.

**Comparison of achievement by high and low engagement (N=928)**
There was no discernable difference between the Likert items (statements rated in the basis of 1 (strongly disagree) to 5 (strongly agree) in the August and November 2013 data (effect sizes very small).

However, there was some evidence of a shift in student perceptions in relation to the more direct questions concerning the five essential feelings included in the November Student survey (n=931 matched pairs) rated on a scale of 1 (strongly disagree) to 10 (strongly agree) (effect sizes small).

<table>
<thead>
<tr>
<th>Thinking about maths …</th>
<th>Aug</th>
<th>Nov</th>
</tr>
</thead>
<tbody>
<tr>
<td>I feel competent</td>
<td>6.6</td>
<td>6.5</td>
</tr>
<tr>
<td>I feel I belong in maths classes</td>
<td>5.7</td>
<td>6.8</td>
</tr>
<tr>
<td>I feel useful</td>
<td>5.6</td>
<td>6.5</td>
</tr>
<tr>
<td>I feel I have choices in maths</td>
<td>5.8</td>
<td>6.7</td>
</tr>
<tr>
<td>I feel optimistic about maths</td>
<td>5.5</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Comparison of mean ratings on CBUPO questions, November 2013 (n=931)
Drawing Task (McDonough, 2002)

*Think of a situation when you are learning maths well. Draw it. Then, describe your drawing*

Responses to this task were requested in both the August and November Surveys. Some interesting trends emerged

<table>
<thead>
<tr>
<th>Initial Category</th>
<th>% Aug</th>
<th>% Nov</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithms or text, no reference to self or others</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>No response, unclear/irrelevant, sad/frustrated</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>Sitting alone in classroom (neutral/happy expression)</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Teacher and self</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>Working with other students</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>Teacher primary focus in classroom</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>Games, manipulatives, real-world</td>
<td>5</td>
<td>13</td>
</tr>
</tbody>
</table>
No response, unclear/irrelevant, sad/frustrated

Just confused and angry with the noise and the maths
A young student is getting frustrated on a maths question, but the teacher can see he/she needs help, so the teacher comes over and helps the stuck child.
Aim: To build a sustainable, evidence-based, integrated learning and teaching resource to support the development of mathematical reasoning in Years 7 to 10

Focus: Identifying the ‘Big Ideas’ in algebraic, spatial and statistical thinking, implementing and evaluating a targeted teaching approach that “covers the curriculum” …

Partners:
- Brisbane Catholic Education Office
- Department of Education (TAS)
- Department of Education and Child Development (SA)
- Department of Education and Training (VIC)
- Department of Education (NT)
- Department of Education and Communities (NSW)*
- Department of Education (WA)*

* SNMY only in 2015
Mathematical Reasoning?

... capacity for logical thought and actions, such as analysing, evaluating, proving, explaining, inferring, justifying and generalising (Australian Curriculum: Mathematics, ACARA, 2015, p. 5)

Mathematical reasoning is not something that students walk into your classroom knowing how to do, and what you expect of them. Rather it is a process that must be learned while you are teaching the content. 

Mathematical reasoning involves much more than asking ‘Why …?” but it is a good start, for example,

• Why is the surface area of a solid cylinder: \(\text{height} \times \text{circumference} + 2 \times \text{area of the base}\) ?

• When solving linear equations, why do you have to do the same thing to both sides of the equation?
For the purposes of the RMFII project:

Mathematical reasoning encompasses:

i. **core knowledge** needed to recognise, interpret, represent and analyse algebraic, spatial, statistical and probabilistic situations and the relationships/connections between them;

ii. **ability to apply** that knowledge in unfamiliar situations to solve problems, generate and test conjectures, make and defend generalisations; and

iii. **a capacity to communicate** reasoning and solution strategies in multiple ways (i.e. diagramatically, symbolically and orally).

**Students are reasoning mathematically when they:**

- explain their thinking,
- deduce and justify strategies used and conclusions reached,
- adapt the known to the unknown,
- transfer learning from one context to another,
- prove that something is true or false; and
- compare and contrast related ideas and explain their choices.

(http://www.australiancurriculum.edu.au/mathematics/rationale)
A Year 8 student’s response to Medicine Doses problem*

\[
\text{Child dose = Adult dose} \times \frac{\text{Age}}{\text{Age} + 12}
\]

(a) If the adult dose for a particular medication is 15 mL, what would be the appropriate dose for a 6 year-old child?

\[
15\text{mL} \times 6 = 90\text{mL} \\
6 + 12 = 18 \div 90 = 5\text{mLs}
\]

(b) A nurse used the formula to work out the dose for an 8 year-old boy. She correctly calculates it as 6 mL. What was the adult dose in this case?

\[
15\text{mL} \times 8 = 120\text{mL} \\
8 + 12 = 20 \div 120 = 6\text{mL}
\]

* Task from Beesey et al (1998), data from MYNRP (Siemon et al, 2001)
**Spatial Reasoning:**

What is involved in solving the following?

O is the centre of a circle of diameter 17 cm.

ABC is a right triangle with the dimensions shown.

What is the length of the line AC?

Find the area of the shaded part if the diameter of the circle is 22 cm and ABCD is a square.
What is involved in solving the following?

A hiker walked 3 km North, 5 km South West then 2 km East. Where was he in relation to his starting point?

Statistical Reasoning:

Ariana had a goal-shooting average of 12 goals before the finals? In the semi-final she scored 18 goals and in the final she scored 15 goals. What was her end-of-season average?

…. connections between related concepts, confidence to use the familiar to develop new ideas (ACARA) …
Proportional Reasoning:

Holiday Destinations

The table shows how many holiday packages were sold to popular destinations in 2013 and 2014 by a travel agency.

The Manager of the travel agency said that the Gold Coast was a more popular holiday choice in 2013 than in 2014. Do you agree with the Manager’s statement? Use as much mathematics as you can to justify your answer.

<table>
<thead>
<tr>
<th></th>
<th>New Zealand</th>
<th>Bali</th>
<th>Gold Coast</th>
<th>Tasmania</th>
<th>Hawaii</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>15</td>
<td>45</td>
<td>36</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>2014</td>
<td>30</td>
<td>72</td>
<td>44</td>
<td>20</td>
<td>34</td>
</tr>
</tbody>
</table>
RMFII Outcomes:

An integrated learning and teaching resource will be developed and disseminated via a web-based portal in collaboration with the Australian Association of Mathematics Teachers (AAMT). This will include:

- validated assessment tools,
- an evidence-based framework for developing mathematical reasoning across the three domains;
- targeted teaching advice for each level of the framework, and
- task-based professional learning modules aimed at deepening teacher’s pedagogical content knowledge for teaching Year 7 to 10 mathematics.
Exploring targeted teaching in NSW using SNMY
One school’s SNMY Results 2015 (used with permission)

Plumpton High School Pre and Post Test Data

Targeted teaching makes a difference
Mathematical Proficiency – the ultimate goal

Strategic competence: Ability to formulate, represent, and solve mathematical problems

Adaptive reasoning: Capacity for logical thought, reflection, explanation, and justification

Conceptual understanding: Comprehension of mathematical concepts, operations, and relations

Productive disposition: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy

Procedural fluency: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

So, what’s the solution?

Targeted teaching directed at a small number of really big ideas in the context of the proficiencies

Big Ideas

- Conceptual Understanding
- Procedural Fluency
- Mathematical Problem Solving
- Mathematical Reasoning
Towards Educating Mathematics Professionals Encompassing Science and Technology

Another AMSPP Project

We are creating a framework for quality mathematics PL

Want to participate in and contribute to the development of quality PL?

TEMPEST will be piloting PL at a place near you

To find out more, collect a flyer on your way out