OPENING DOORS TO SUCCESSFUL NUMBER LEARNING FOR THOSE WHO ARE VULNERABLE

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Finding the right key for opening the door to successful number learning for every child can be a challenge. However, there are a few master keys that open doors for many. This paper explores the challenges of successful number learning and teaching, with a particular focus on opening doors for those who are vulnerable. Learning keys examined include a focus on quantity, mental number lines, place value and reasoning strategies for calculating. Teaching keys considered include understanding pathways of number learning, common difficulties, customizing learning experiences based on children’s current knowledge, and having a whole school approach.

Vulnerability and building bridges

There is strong community endorsement throughout Australian society for mathematics being an important aspect of children’s education and futures. School communities, therefore, accept an awesome responsibility to provide successful learning experiences for all students. This is a demanding, but achievable goal. Finding the right key for opening the door to successful number learning for every child is a challenge. However, there are a few master keys that open doors for many. Examining these master keys, particularly in relation to number learning, is what this paper is about.

Some children appear to be vulnerable when it comes to number learning. Possible reasons include differences between children’s early environments (including factors such as culture, language, attitudes to education, nurturing, and family harmony) and the school environment (including socio-cultural norms such as organisational structures, communication, expectations and attitudes of teachers, and school expectations of behaviour), ill health (including physical disabilities), learning disabilities, and discrepancy between a child’s approach to learning and the teaching style a child encounters. The presence of these factors in a child’s environment may require particular responses from teachers to ensure successful mathematical learning for children. It is likely that some children may need more customised approaches than the traditional classroom program provides. Thus, it is important that schools are able to respond to vulnerable learners by providing customised programs and support.

Another reason for some children’s vulnerability is the difference between formal curriculum content in mathematics and children’s informal mathematical knowledge. This may delay learning in formal schooling contexts (Doig, McCrae, & Rowe, 2003) and increase vulnerability. Griffin & Case (1997) found that a gap existed, at the very start of schooling, between the informal understandings children have available to make sense of mathematics and the formal learning opportunities they are exposed to in schools.
Such findings have lead to calls for reform and the development of new curriculum (Griffin & Case, 1997).

Children’s informal mathematical knowledge is sometimes culturally specific, and may not be obvious to the teacher. Also, some children may not have the chance (or language) to demonstrate their informal knowledge in the context of a formal mathematics program. Thus, the first master key for successful learning is a teacher who is able to create a bridge for children as they negotiate the transition from home environments to the more formal world of the school and learning school mathematics.

**Focus on quantity**

Understanding quantity another master key for successful learning, yet this is difficult for some children to achieve. Indeed, although such children may be able to successfully read and write numerals (and therefore may ‘look’ like they understand numbers), they may not have connected the words and symbols to the actually quantity each represents. Indications that children understand *quantity* include them being able to:

1. Quantify collections (especially for more than 30 or 100 items). This requires children being able to explain why one-to one correspondence is important for proving the quantity of items in a collection, and why strategic groupings of items (such as grouping into tens) may be useful.  
2. Explain what is one more or one less (or ten more/ ten less or 44 more / 44 less etc.) than a given quantity, and how they might prove this to another person. 
3. Partition quantities in multiple ways (e.g., $30 = 15 + 15 = (2 \times 10) + 8 + 2 = (3 \times 10) = 7 + 6 + 17$).  
4. Subitise quantities (e.g., recognising visual patterns such as those found on dice, dominos, playing cards and ten frames, and environmental patterns (quantity of petals in a flower, trees on the horizon, birds in a flock etc).  

Children who are able to work with quantities in these ways are well prepared for the challenges of mathematics learning at school. Teachers need to provide opportunities for such learning to take place for all children.

**Development of a mental number line**

Another important learning master key is forming a *mental number line*. This requires the ability to visualise and abstract a number line so that you can order numbers by quantity, locate any given number along the number line, and generate any portion of the number line that may be required for problem solving. Children’s mental number lines continuously increase as they encounter and construct concepts of new numbers (e.g., millions, trillions, fractions and decimal fractions).

A mental number line is called on when we try to locate a house along a street according to its street number, or read an analogue clock without numerals, or attempt to find a number on a Tattslotto ticket, or a numeral on a game board. Griffin, Case, & Siegler (1994) highlighted three important reasons for focusing on children’s development of a mental number line: (a) teaching children to respond to questions about relative magnitude in the absence of any concrete sets of objects, (b) teaching children the increment rule, i.e., the addition and subtraction of one element alters the cardinal value of the set by one unit, and therefore moves the value one unit up or down on the number line, and (c) teaching children that knowledge of relative position on the number line...
line is useful for determining relative quantity in various tasks, when it cannot be determined more directly.

One way to help children develop a mental number line is to engage them in activities involving an *empty number line*. This is a strategy widely used in the Netherlands where they have adopted a curriculum based on the empty number line. “The choice of the empty number line as a linear model of number representation up to 100 (instead of grouping models like arithmetic blocks) reflects the priority given to mental counting strategies as informal knowledge base” (Beishuizen & Anghileri, 1998, p. 525). This was due to Freudenthal, who in opposition to the ‘New Maths’, advocated linking early mathematics activities to children’s own informal counting and structuring strategies, and that the “discovery of simple patterns and easy structures like abbreviated counting in steps of twos, fives and tens was conceived as an important emergent mathematising activity” (Beishuizen & Anghileri, 1998, p. 523). The Dutch emphasis is on children’s own informal methods, but the development of more sophisticated strategies is not left to chance. Learning opportunities are deliberately designed and sequenced to invite specific strategies and to stimulate abbreviation towards higher level strategies. The idea is that counting strategies should not be suppressed, but mastered.

The development from informal strategies to higher level formal strategies is achieved through cognitive and metacognitive action: monitoring strategy selection by written recording and reflection on strategy choice in whole class discussion (Beishuizen & Anghileri, 1998). Until this is well established, the Dutch advocate that a focus on partitioning numbers into tens and ones is delayed. However, once children have established methods for partitioning numbers, it is important that this knowledge is integrated with counting strategies.

Once children begin to partition numbers into tens and ones, another issue to focus on is ensuring that children construct abstract concepts of ten as both 10-ones and 1-ten that can be counted and decomposed, traded and exchanged. This knowledge needs to extend to concepts of 100 and 1000, and so on, that can be counted and decomposed, traded and exchanged. Most importantly, children need to understand how numbers may be partitioned to enable efficient calculations for complex problem solving. In many ways, this depends on children’s ability to partition numbers flexibly.

**Using reasoning strategies for calculating**

Many older children who remain mathematically vulnerable have become reliant on using *count-on* strategies when calculating. Although this is an effective strategy for producing answers, it is inefficient and time-consuming for complex calculations. Indeed, the amount of effort required for counting-on or counting-back in complex calculation contexts detracts from children’s opportunities to focus on the mathematical concepts that may be the focus of a lesson. For example, imagine a lesson designed to enable children to explore the concept of *average* in which they needed to explore a set of data (11 9 9 9 12 9 9 8 9 9 5 9 9) and determine the *mean*. If some children are reliant on *counting-on* as an addition strategy in this situation, then it is possible that they would expend so much effort performing the calculation (possibly by following a set procedure), that there would be little opportunity to notice the *central tendency* of the data and therefore construct new knowledge about *means*. However, if a child was able to perform the calculations through using reasoning strategies, they would be more likely to notice the individual data and its tendency to centre around the mean.
The message of this story is that it is important for children to develop reasoning strategies not only so that they can perform calculations efficiently, but also so that they may have the opportunity to fully engage in mathematics learning experiences where calculation is incidental to the mathematical focus of the lesson. A focus on children developing reasoning strategies is therefore important. Indeed, I believe that an important educational goal is for children to be using reasoning strategies by the end of their third year at school, before counting-on and counting-back become entrenched strategies.

Reasoning strategies include using doubles and near-doubles, recognising number partners for ten (and 20/50/100 etc.), building to the next ten, using commutativity principles, using inverse relationships (e.g., using an addition fact to solve a subtraction calculation), adding 10, and compensation strategies, such as adding 9 by adding 10 and subtracting one. Thus an important master key for successful learning is assisting children to become aware of, understand and be confident to use these reasoning strategies.

Understanding pathways of number learning

A teaching challenge is matching children’s learning experiences to their individual learning needs. Thus, an important master key for opening the door to successful learning for all children is for teachers to understand learning pathways so that they can customise children’s learning experiences to match each child’s position along these pathways. One model for describing pathways of children’s number learning is the Growth Points developed during the Early Numeracy Research Project (ENRP, Clarke et al., 2002). These Growth Points (See Appendix A) are used to describe the pathway of children’s learning in Counting, Place Value, Addition and Subtraction and Multiplication and Division. Teachers can determine the growth points children have reached in each of these domains by using the Early Numeracy Interview (Department of Education Employment and Training, 2001), and through subsequent assessment and observation during lessons.

Once a child’s Growth Point is known, then teachers can determine the zone of proximal development for that child’s learning and design appropriate instruction. It is also important to know the challenges involved in reaching the next growth point so that they may identify the aspects of learning experiences to draw attention to so that children may construct the knowledge required to reach subsequent growth points.

Customising learning experiences

The next master key for opening doors to successful learning is teachers customizing learning experiences based on children’s current knowledge (or Growth Points). To enable this to happen, it is important for teachers to structure frequent opportunities to observe each child’s problem solving so that children’s strategies, errors and misconceptions may be noted and used to plan and customise subsequent learning opportunities. Next teachers must implement teaching strategies and learning activities that can be customised to enable a focus on children as individuals, and that engage children as individuals. For this purpose, both open tasks and games can be useful. But the secret to using such activities is the strategic presence of the teacher during the activity so that they may interact with each child to help each to notice the aspects of the activity that will lead to the construction of mathematical knowledge, and to customise the experience to meet the learning needs of each child. This does not mean that the
teacher needs to be with each child all of the time, but the teacher does need to be roving to observe each child’s strategies and to provide strategic scaffolding, questioning and activity adjustments when necessary so that learning opportunities for each child are optimised.

For example, suppose that a child has reached Growth Point 4 in Counting and is therefore able to recite counting sequences in tens, fives and twos when beginning at zero. Identifying the next growth point for the child along the learning pathway means that the teacher understands that the next major step in the child’s learning is being able to count by tens, fives and twos from any starting point in the number sequence. Knowledge of the challenges inherent in reaching this growth point (children understanding the quantity associated with each number name, and understanding that the quantities increase or decrease by ten, five or two each time, according to the count) help to guide teacher’s selection of learning experiences and interactions with students. For example, teachers may provide opportunities for children to count large collections of items grouped in tens, fives and twos to emphasise the quantities involved and also ask children to use an empty number line to keep track of the count. Through questioning, the teacher may help the child to notice that the quantities are increasing or decreasing by the same amount each time, and also to notice the pattern associated with the numerals used to record the quantities. Teachers can customise this type of activity according to whether the learning focus for a child is counting by tens, or fives, or twos or sevens or five-tenths or ones. These types of experiences, interactions and any associated reflection will help each child construct the knowledge associated with the next growth points along their learning pathway.

**Open tasks**

Open tasks also provide useful contexts for customising learning opportunities. However, it is important not only to develop the initial task, but also to develop alternatives for those who experience difficulty, and for those who require extension. A counting task that has been designed in this way is shown in Figure 1. The lesson format for using open tasks involves: (1) presenting the task and clarifying its meaning and goals; (2) making it explicit that children are required to find multiple solutions and to discuss ideas with each other; (3) working on the task for an extended period; (4) teacher roving to observe children’s strategies, interact with students, and adjust the task to match the learning needs of individuals; (5) extended class discussion of solutions and strategies; and (6) teacher summary of the key mathematical ideas.

<table>
<thead>
<tr>
<th>Open Task for Investigation</th>
<th>Alternative to prompt children having difficulty engaging</th>
<th>Alternative to extend children further</th>
</tr>
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<tbody>
<tr>
<td>Nuno was using the calculator (constant function) to count by fives. I couldn’t see the whole calculator display, but I noticed the last digit in the display was 7. What could be the next 4 numbers that the calculator shows?</td>
<td>Nuno was using the calculator to count by twos. I couldn’t see the whole calculator display, but I noticed the last digit in the display was 7. What could be the next 4 numbers that the calculator shows?</td>
<td>Nuno was using the calculator to count by sevens. I couldn’t see the whole calculator display, but I noticed the last two digits in the display were 44. What could be the next 4 numbers that the calculator shows?</td>
</tr>
</tbody>
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*Figure 1. An Open Task with alternatives for simplifying and extending the task.*
Having a strategic whole school approach and providing intervention programs

It is important that school communities strategically use what is known about number learning and children’s difficulties to address the perennial problem of the presence of children in classrooms who have difficulty learning mathematics. I propose that the key to preventing children from experiencing difficulty in mathematics lies in using assessment information to identify children who are vulnerable in number learning (Gervasoni, 2004b), and in utilising knowledge about the common difficulties that children experience so that curriculum experiences and instructional practices may be designed and implemented to address these difficulties. It is anticipated that such an approach will involve two main instructional waves: the first involving classroom teaching that strategically addresses difficult aspects of number learning and rigorously supports children through the learning process; and the second wave involving specialist teacher support and intervention programs for children who need more intensive instruction and accelerated learning opportunities.

The optimal time for children to participate in an intervention program is during their second year at school, and before they experience a sense of failure. Older children may also benefit from additional assistance programs, but the older they are, the more complex and longer term their needs for assistance become. Thus a strategic whole school approach involving assessment, school organisation, school program co-ordination and development, intervention programs, student and family welfare support, and professional development is another master key for ensuring successful learning for all students.

Strategically addressing common difficulties in the early years

The type of difficulties encountered by young children who are vulnerable in aspects of number learning have been well documented (Gervasoni, 2004a). Classroom teachers who are aware of these difficulties can design curriculum and instruction aimed at preventing children from experiencing these difficulties in the first place. Further, school communities can introduce assessment and monitoring strategies to identify any children who experience these difficulties and provide more intensive instruction to assist them. Thus an important master key for opening doors to successful learning for all children involves implementing the following steps in the early years of schooling:

1. Acknowledge children’s informal mathematical knowledge and cultures by creating a bridge for children as they negotiate the transition from their home environment to the more formal world of the school and learning school mathematics.
2. Structure frequent opportunities to observe each child’s problem solving so that children’s strategies, errors and misconceptions may be noted and used to plan and customise subsequent learning opportunities;
3. Implement teaching strategies and activities that can be customised to enable a focus on children as individuals, and that engage children as individuals;
4. Provide experiences that focus children’s attention on the nature of the number sequence and that number words relate to collections. In particular, help children to notice that when counting by ones, each successive number in the sequence has a value of one more or one less than the previous number. Provide opportunities...
for children to count collections of objects so that they are producing collections that are one item larger or smaller with each action, and can link this phenomenon with the number word representing the collection after each action. It is important that the number sequence produced when counting is meaningful and understood by children, and that eventually children can generalise to produce any part of the number sequence. For this to occur, a mental image of the number sequence is important, and helping children to form this mental image is important;

5. Provide experiences that focus on the meaning of the decade number words, and the patterns inherent in number names. Grouping objects in tens and ones when object counting will assist children to notice the place value structure underpinning number names and numerals, and will assist children to understand the decade transitions. Also, address the similarity in sound between ‘ty’ and ‘teen’ and assist children to understand what the differences in the ‘sounds’ mean;

6. Assist children to notice the principles that underpin successful object counting, and that when counting large collections of objects, the objects need to be managed to distinguish those that have been counted from those that have not been counted;

7. Provide opportunities for children to count backwards, initially with objects to model the number names, but importantly to draw children’s attention to the nature of the sequence, and that it is the same sequence produced when counting forwards;

8. Provide opportunities for children to construct a concept of ten as both 10 ones and 1 ten that can be counted and decomposed, traded and exchanged. This knowledge needs to be linked to the place value conventions used to read and write two-digit numbers and larger numbers. Also ensure that children notice that the face value of digits in a numeral represents the cardinal value of a group;

9. Provide opportunities for children to construct abstract reasoning strategies and visualisation for solving problems. This is particularly important for children prior to Grade 2, because some become reliant on modelling and counting strategies. To avoid this situation, children need to be carefully monitored and provided with intensive instruction where necessary. In particular, children need more experiences that focus on subtraction situations; and

10. Provide experiences that focus children’s attention on the grouping structures inherent in Multiplication and Division tasks, and strategies such as skip counting and number combinations that use the group structure to solve problems.

11. **Conclusion**

Finding the right key for opening the door to successful learning for every child is an important goal of education, but a complex task. However, there are a few master keys that open doors for many. In summary, some master keys that will help all children learn mathematics successfully are: (a) providing opportunities for all children to construct powerful schema involving quantity, a mental number line, and reasoning strategies for performing calculations; (b) understanding the pathways of mathematical learning and how to customise learning experiences based on each child’s current knowledge; (c) strategically addressing the known common difficulties experienced by young children early in their schooling; and (d) providing a whole school approach to improving mathematical learning for all students, including the provision of intervention programs.
for those who are most vulnerable. Using these master keys is one way that school communities can prepare children for successful futures.

References
Appendix A: Early Numeracy Research Project
Assessment Framework

Number Growth Points

(February 2001 Version with CEO Ballarat 2002 Extension)

Notes:

- Growth points are not necessarily hierarchical, but involve increasingly complex reasoning and understanding.
- It must be emphasised that conclusions drawn in relation to placing students at levels within this framework are based on a 30-minute (approx.) interview only. Ongoing assessment by the teacher during class will provide important further information for this purpose.
- Student understanding may be reported as a “0”. This should not be taken as an indication of “no knowledge” or “no understanding”, but rather as an indication of a lack of evidence of “1”.
- The Catholic Education Office Ballarat Diocese is trialling a set of extension growth points for Counting, Place Value, Addition and Subtraction, and Multiplication and Division. These are recorded below also.

A. COUNTING
0. Not apparent.
   Not yet able to state the sequence of number names to 20.
1. Rote counting
   Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.
2. Counting collections
   Confidently counts a collection of around 20 objects.
3. Counting by 1s (forward/backward, including variable starting points; before/after)
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before after a given number.
4. Counting from 0 by 2s, 5s, and 10s
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from x (where x >0) by 2s, 5s, and 10s
   Given non-zero starting point, counts by 2s, 5s, 10s to given target.
6. Extending and applying counting skills
   Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

CEO Extension
   Counting by fractions and decimal fractions
   Can count from a non-zero starting point by fractions and decimal fractions.

C. STRATEGIES FOR ADDITION AND SUBTRACTION
0. Not apparent
Not yet able to combine and count two collections of objects.
1. Count all (two collections)
   Counts all to find the total of two collections.
2. Count on
   Counts on from one number to find the total of two collections.
3. Count back/count down to/count up from
   Given subtraction situations, chooses appropriately from strategies including count back, count down to & count up from.
4. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)
   Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.
5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)
   Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.
6. Extending and applying addition and subtraction using basic, derived and intuitive strategies
   Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.

CEO Extension
Adding and subtracting fractions and decimal fractions
Given a range of tasks involving fractions and decimal fractions, can solve them mentally or on paper, using the appropriate strategies and a clear understanding of key concepts.

B. PLACE VALUE
0. Not apparent
   Not yet able to read, write, interpret and order single digit numbers.
1. Reading, writing, interpreting, and ordering single digit numbers
   Can read, write, interpret and order single digit numbers.
2. Reading, writing, interpreting, and ordering two-digit numbers
   Can read, write, interpret and order two-digit numbers.
3. Reading, writing, interpreting, and ordering three-digit numbers
   Can read, write, interpret and order three-digit numbers.
4. Reading, writing, interpreting, and ordering numbers beyond 1000
   Can read, write, interpret and order numbers beyond 1000.
5. Extending and applying place value knowledge
   Can extend and apply knowledge of place value in solving problems.

CEO Extension
Reading, writing, ordering and interpreting millions, fractions and decimal fractions
Can read, write, interpret and order numbers beyond millions, and with fractions and decimal fractions.

D. STRATEGIES FOR MULTIPLICATION AND DIVISION
0. Not apparent
   Not yet able to create and count the total of several small groups.
1. Counting group items as ones
   To find the total in a multiple group situation refers to individual items only.
2. Modelling multiplication and division (all objects perceived)
   Models all objects to solve multiplicative and sharing situations.
3. Abstracting multiplication and division
Solves multiplication and division problems where objects are not all modelled or perceived.

4. Basic, derived and intuitive strategies for multiplication
   Can solve a range of multiplication problems using strategies such as commutativity, skip counting and building up from known facts.

5. Basic, derived and intuitive strategies for division
   Can solve a range of division problems using strategies such as fact families and building up from known facts.

6. Extending and applying multiplication and division
   Can solve a range of multiplication and division problems (including multi-digit numbers) in practical contexts.

CEO Extension
   Multiplying and dividing fractions and decimal fractions
   Given a range of tasks involving fractions and decimal fractions, can solve them mentally or on paper, using the appropriate strategies and a clear understanding of key concepts

AAMT STANDARDS: 1.1, 3.3

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