DIFFERENT APPROACHES: DIFFERENT OUTCOMES

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Much change has taken place from the traditional mathematics classroom typified in the 1950’s to that of today. For example, classroom activity today takes into account the importance of classroom discussion and pupil’s use of mathematical language, activities that are hands-on and connected to the mathematics used in real life, and the provision of opportunities for students to reflect on the mathematics they are coming to understand. The way in which teachers handle classroom discussion has become paramount to enhancing and extending children’s meaningful understanding of mathematical concepts.

The Traditional Mathematics Classroom

There are many descriptions of the ‘traditional mathematics classroom’. Sullivan, Bourke & Scott (1995) give the following scenario for what they term ‘conventional teaching’:

Consists of teacher demonstration of one or more exercises, with explanations and demonstrations linked to examples. Students predominantly work on drill and practice exercises. If the feedback is negative, students do more practice; if the feedback is positive then the class moves onto new exercises. Students are likely to see mathematics as a collection of rules and exercises. (Sullivan, Bourke & Scott, 1995, page 484)

Prawat (1992) summarises the approach in an appropriate way by claiming that traditional practice is best characterised as a 'transmission' approach to teaching where teachers are 'tellers of the truth', and an 'absorptionist' approach to learning where learners act out a role of 'accumulators of material who listen, read, and perform prescribed exercises' (page 356).

Personal Classroom Memories

I was a primary school student in the 1950’s and can clearly remember many aspects of the mathematics (arithmetic) teaching that I received. I can remember:

• sitting in rows of desks with everyone facing the front of the classroom;
• being given teacher demonstration of a process and then ‘practicing the process’ where the ‘practising’ was done in complete silence (in fact I was given the strap once for talking in arithmetic!);
• that some of the exercises involved lengthy calculations, for example, how many inches in 2 miles, 40 chains and 10 yards? – these conversions often involved working through the whole gamut of measures: furlongs, chains, rods, links, yards, feet and finally inches;
• the end-of-month test where the results determined where you sat in class – the top-scorer sat at the back in row 1 and the lowest-scorer sat in the front desk of row 5 with the rest of the class arranged in order between these two students;
• that even though we were arranged in order or grouped according to ability the whole class did the same work from the blackboard;
that we sometimes did ‘practice’ examples from the Education Department book *Arithmetic for Grade VI* and when the ‘practising’ was complete there was often a real world problem to hone our skills (see figure 1)

Fortunately for me at that time I loved doing these exercises and working in this way (although I didn’t enjoy getting the strap), and received good grades at the end of the year. Other year 6 memories include very practical activities such as nature walks along Gardiner’s Creek collecting grass seeds and observing the oxbow lakes and bank erosion. Why wasn’t mathematics learning like this? I imagine that it is tied up with beliefs about the nature of mathematics and how mathematics was best taught in those days. And a class of more than 60 students would not have been conducive to the innovative approaches to teaching that are used today. As far as I know all the students loved this grade 6 teacher and he was well respected by our parents.

**Calls for Change**

Since the early 1980s there have been repeated calls for change to mathematics instruction.

We are convinced that school mathematics in virtually all countries around the world has been tried in the balance and found wanting. Fundamental changes are required: there needs to be a reconceptualisation of what school mathematics should be about (the “why”), what mathematics should be studied in schools (the “what”), how it should be presented and how it should be assessed (the “how”). (Ellerton & Clements, 1989, page vii)

Sullivan (1989) noted that there were two major calls for change: firstly that pupil understanding needs to be paramount, and secondly that pupils need to see that the mathematics they are learning is relevant to everyday life. He pointed out that there is a gap between recommendations from the theory of mathematics instruction and apparent classroom teaching. He cites a number of examples of apparent practice that the ‘theorists’ criticize. Some examples include: relying on short term goals (Skemp, 1971); over-emphasizing irrelevant formalism and rote learning (Willoughby, 1983); and presenting content which is divorced from reality or any context (Hoyles, 1985; Lowe & Stephens, 1987) (Taken from Sullivan, 1989, pages 3 & 4).

**What Does the Mathematics Education Literature Suggest for Today’s Teachers?**

Gervasoni (1995) analysed a number of documents and professional development course outlines to ascertain what recommendations were being made for the advancement of mathematics instruction in Victoria. She noted the following ‘images’:

A mathematics teacher

(a) helps students develop positive attitudes towards mathematics;
(b) helps students understand that learning involves taking risks;
(c) utilises constructivist learning approaches;
(d) challenges students' existing conceptions;
(e) provides opportunities for students to reflect on their learning;
(f) provides feedback to students about their mathematical learning;
(g) provides challenge within a supportive framework;
(h) utilises co-operative learning models;
(i) develops activities which are purposeful, interesting, and build upon and respect students' life experiences;
(j) gives students the opportunity to talk and write about mathematics;
(k) uses gender and culturally inclusive language, resources and activities;
(l) involves students in problem-solving activities;
(m) enables students to experience the process through which mathematics develops; and
How do these ‘images’ play out in today’s classroom? The following two sections explore this question. Firstly, three classroom activities are described with connections made to Gervasoni’s ‘images’. And secondly, discussion centers on how teachers handle classroom discussion in relation to Gervasoni’s ‘image’ relating to utilizing constructivist principles [point (c)].

Gervasoni’s ‘Images’ in Classroom Activities

Secret Spinner

Secret Spinner (Smith, 2003) is a classroom activity that fulfils many learning outcomes from the CSFII Number and Chance & Data strands. For this activity the teacher places a spinner unknown to the students in a container or box lid. The teacher explains to the students that they will collect data from successive spins in order to predict what the colour combinations on the spinner might be. The teacher completes about ten spins with the students recording the results. The students are then asked to draw what they think the spinner looks like and share their ideas with a partner.

A further ten spins are completed, the students adding these results to their original data. Students are then asked to re-visit their initial prediction/drawing and draw a new spinner if warranted. The teacher facilitates further sharing and discussion as a whole class. Finally, the spinner is shown and comparisons are made with the students’ predictions.

During this whole class discussion it is important to listen to the language the students use. I have had year 1 students using fraction language in an appropriate way and year 4 students confidently describe their predictions using percentage. On the other hand a year 4 student described their spinner as being 1/2 blue, 3/4 green and 3/4 yellow. This student obviously needs some additional assistance with early fraction knowledge.

When completing this task in grades ranging from year 1 through to year 6 it has been interesting to note how the students complete the recording of the data. Some students write down the full colour word each time and consequently have a listing of unorganized data. Whereas other students may keep a tally graph. When facilitating classroom discussion it is worth having students share their ways of recording and hopefully other students will adopt more efficient and useful ways when the task is repeated.

This classroom activity fulfils some of Gervasoni’s ‘images’, including:

(b) helps students understand that learning involves taking risks;
(c) utilises constructivist learning approaches;
(j) gives students the opportunity to talk and write about mathematics;
(l) involves students in problem-solving activities;
(n) uses a variety of assessment strategies and records.

Tell & Sign Activity – Place Value

For this activity each student needs a grid sheet (see Figure 2) and a hands-on teaching aid (allow the students to choose from a pack Montessori Cards, a set of stick bundles, or an activity board with ten tenframes and some counters). This specific activity meets place value learning outcomes at a year 2 level.
The students talk to each other in pairs. Student A chooses a number from the grid sheet and shows student B that number modelled using their teaching aid and explains how the number can be extended to separately show the tens and the ones components. Student A initials student B’s grid sheet in the box containing the modelled number. Student B then models and gives an explanation to student A and signs the appropriate box on students A’s grid sheet. The two students move on to new partners, only being allowed to model one number to any partner. By the end of the activity the students will have collected up to twelve different initials on their grid sheet.

This activity could be used as pre-assessment or post-assessment in a series of lessons on place value. Teachers should participate in this activity in order to collect anecdotal assessment on individual pupils and to inform their lesson planning.

During this activity the students see the two-digit numbers modelled in various ways and hear differing explanations for extending two-digit numbers. This encompasses Diene’s notion of multiple embodiment – gaining different visual and mental images of the same concept.

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  (g) provides challenge within a supportive framework;
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  (n) uses a variety of assessment strategies and records.

**Pick a Box**

Pairs of students need the 4 Bs activity board (see figure 3) and a set of numeral cards for the numbers 1 to 20. Student A shuffles the cards and then places four cards on to the activity board, one card in each picture, all face up except for the one placed in the box. The number on the card placed in the box becomes the ‘mystery number’ for student B to work out. Student B asks questions of student A in order to work out the ‘mystery number’ but each question must contain one of the visible numbers. Student A needs to peep at the ‘mystery number’ in order to be able to answer Student B’s questions. Student A answers either ‘yes’ or ‘no’. A tally is kept of the number of questions asked and that becomes Student B’s score. The students swap roles. The student with the lowest score is the winner (Smith, 2004).

Assume that the visible numbers are ‘3’, ‘12’ and ‘20’. Student B’s questions could include:
Is the number less than 12?
Is the number more than 12 + 3?
Is the number odd like 3?
If the ‘mystery number’ is ‘16’ then Student A’s response to the above questions would be ‘No’, ‘Yes’, ‘No’. These three questions and their responses would enable Student B to work out that the ‘mystery number’ was either ‘16’ or ‘18’. Student B could then ask a more specific question such as, ‘Is the ‘mystery number’ 20 take away 4?’.

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(j) gives students the opportunity to talk and write about mathematics;
(l) involves students in problem-solving activities.

**Handling Classroom Discussion**

The outcomes of any classroom activity often depend on the way in which the teacher handles classroom discussion. The following extracts are taken from three different classrooms that were observed as part of a research project (Smith, 2001).

**Karen’s Location Lesson**

Karen was teaching year 3 and she had placed the letters A – J across the front of the classroom and the numbers 1 – 15 along one of the side walls. The students had to work out where they were sitting by visualizing a ‘Melways-type’ grid in the classroom. For example, Geraldine thought that she was sitting at ‘B6’. An incident occurred when several children where clearly sitting in the same row but each one of them had recorded a different number. Karen used this classroom incident to facilitate the following discussion:

Karen: OK we have a problem here. It looks like they’re at the same number and yet they have written down something different. How might that have happened?

Pupil: Jenny might have moved.

Karen: She could have moved her chair a little bit. How could we be more accurate?

Pupil: Get a big ruler.

Karen: That’s a very good idea ….. we could get a big ruler and rule lines across the carpet. Yes Roma?

Roma: Walk there.

Karen: Walk there. Walk in a straight line.

Annie: Lie down.

(Smith, 2001, p. 131)

**Deidre’s Subtraction Lesson**

Deidre was also teaching year 3 and she was demonstrating how to set out the formal process for subtraction.

Deidre: Now I want you to set them out like this. [Deidre wrote on the board a vertical format for 38 - 26] I’ve done it like this. It makes it easy. You can see which column’s in which. Shirley what do I have to do first?

Shirley: Take the 2 away from the 3.

Deidre: Do I? If you’re doing any sort of maths which column do you always start with, Trixie?

Trixie: Units.

Deidre: Always start with the units column and work that way. If you go back from this way to that way ….. that’s right you can’t go carry.

(Smith, 2001, p. 130)
**Gerry’s Map Lesson**

Gerry was teaching a year 4/5 composite class. They were about to attempt a worksheet involving totaling distances on a map. Eventually the classroom discussion focused on the notion of scale.

**Gerry:** Well what we’re after is what the word scale means. When I say that it’s either to scale or not to scale ….

**Pupil:** It’s a balance.

**Gerry:** Yes we’re balancing with a scale.

**Pupil:** Measuring.

**Gerry:** Measuring, right. When you actually look at a map the map is not the same size as the actual area, is it? You’re not going to have a map that’s 32km thick. We’d never be able to have a map like that. So we draw maps to scale. So for example ….

**Pupil:** It’s smaller.

**Gerry:** You make it smaller, don’t you?

(Smith, 2001, p. 134)

**Comment – Handling Classroom Discussion**

Gervasoni listed as one of the ‘images’ the notion of ‘utilising constructivist learning approaches’. Cobb (1986) denotes one form of constructivist theory as ‘radical constructivism’ and describes this as a view where ‘learning is a problem-solving process in which the learner attempts to overcome obstacles or contradictions that arise as he or she engages in purposeful activity’ (Cobb, 1986, page 302). Zevenbergen, Dole & Wright (2004) pick out the following three underpinning premises for constructivism:

Rather than being passively received, students actively construct knowledge.

Mathematical knowledge is created by students as they reflect on their physical and mental actions … [thus] integrating new knowledge into their existing mathematical schemas.

Learning mathematics is a social process where, through dialogue and interaction, students come to construct more refined mathematical knowledge. (Zevenbergen, Dole & Wright, 2004, p. 23)

Zevenbergen, Dole & Wright (2004) stress that dialogue and argumentation are central to a constructivist learning environment. This implies that the way in which teachers handle classroom discussion is paramount to student learning. Classroom discussion includes questioning and giving explanation. Both teacher and student can involve themselves in both of these aspects. Do Karen, Deidre and/or Gerry present a picture of a teacher facilitating discussion along constructivist approaches? Karen is very accepting of her student’s ideas although she expands further the student’s suggestions with her own interpretation rather than by further probing the student. Deidre gives her own explanation of the ‘rules’ and any questioning is very closed. Gerry gives meaningful explanations as student prompts are given but does not allow the students to give the explanations and actually never asks a question. Teachers need to reflect on the balance they give to teacher questioning versus pupil questioning as well as teacher explanation versus pupil explanation.
In Conclusion

Skemp (1976) refers to two differing views of understanding – instrumental and relational. Skemp describes his instrumental view as ‘rules without reason’ (page 20) whereas his relational view he sees as ‘knowing both what to do and why’ (page 20). Skemp’s notion of instrumental understanding of mathematics is based upon a view that sees mathematics as a collection of many disconnected rules, that specific methods match specific problems, and an awareness of the lack of connection between mathematical ideas. His notion of relational understanding allows for connections to be made between fundamental ideas and sees that mathematical principles have broad application. My 1950’s experience of learning mathematics described earlier fits Skemp’s instrumental view whereas the ‘images’ located by Gervasoni tie in with relational understanding. When teachers reflect on their teaching they need to ask, ‘Did the way I planned and implemented the lesson and the way I handled classroom discussion enhance relational understanding for my students?’ Pointers for answering this question are in Gervasoni’s list of ‘images’.

As teachers take on board innovative and creative approaches to teaching, classrooms are looking very different to my 1950’s experience. Students sit in small groups not necessarily all facing the front of the classroom. Talk is allowed and even encouraged. Using hands-on material to model mathematical situations has become important. Connecting mathematics learning to what else happens in the classroom and to real life experiences is assumed essential. Teachers facilitate a ‘discourse community’ where student ideas are accepted, valued and further developed. And having class sizes half of that in the 1950’s has helped these innovations to happen.

References


Education Department, Victoria (1941). Arithmetic for Grade VI. Melbourne: Education Department, Victoria


