What are the characteristics of highly effective teachers of numeracy? Which of these are features of your teaching of mathematics? Which might you work towards developing? Drawing on a range of perspectives, most particularly research findings from case studies within the Early Numeracy Research Project (ENRP), characteristics of highly effective teachers of numeracy are discussed. It is suggested that the characteristics of these teachers are relevant to, and can inform the teaching of mathematics in the early years and at other levels.

Introduction

Teachers don’t merely deliver the curriculum. They develop, define it and reinterpret it too. It is what teachers think, what teachers believe, and what teachers do at the level of the classroom that ultimately shapes the kind of learning that young people get. (Hargreaves, 1994, p. ix)

In working with preservice teachers I stress that it is they who make the decisions as to what they will do and how they will do it when teaching a mathematics lesson. That is, for example, they make the decisions of what words to use when introducing a task, what interactions to have with individual children, or which children’s responses to focus on, and exactly how to utilise these,
during lesson share time. Even when teachers plan with others or follow a lesson from a published resource, they each make what might be called the fine-grain decisions of teaching. Research conducted by Boaler (2003) indicated that ‘it is not the fact that students work in groups, or listen to the teacher, that is important, it is how they work in groups, what the teacher says, and how the students respond [that is important]’ (p. 6). This suggests that when following a lesson procedure, or perhaps even utilising a quality game, the work of teaching can look very different, and thus a common structure cannot guarantee a highly effective mathematics lesson. Individual teachers each play a key role in determining the nuances of each lesson and therefore in contributing to their children’s growth in mathematical understanding.

As teaching is a complex process that involves many elements, it is important that teachers think carefully about what they do and the way they do it, that is, about the fine-grain decisions they make when teaching. In reflecting on teaching it is worthwhile to consider what is known about effective teachers of mathematics. In this paper I examine a range of findings of the characteristics of effective teachers of mathematics. Most consideration is given to the characteristics of highly effective teachers of numeracy as identified within the Early Numeracy Research Project (ENRP), a major study of the teaching and learning of mathematics within the first three years of schooling, conducted in Victoria from 1999 to 2001 (Clarke, 2001). The reader is encouraged to reflect upon these findings in light of their own practices whether a teacher of mathematics in an early years classroom or beyond.

Beliefs about the characteristics of highly effective teachers of numeracy

Imagine you have the opportunity to sit in on a numeracy session of a teacher who is highly effective in helping children grow in their mathematical understandings, and you observe, listen and talk to this teacher. What do you believe would become apparent to you as the characteristics of this highly effective teacher? That is, what do you believe are the characteristics of highly effective teachers of numeracy? You are encouraged to jot down your ideas.

In responding to this question, you might think first about what you believe is the meaning of effective. How might this be judged? What criteria might you take into account? Consider then the role of the teacher within an effective
numeracy lesson. For example, what teacher knowledge, expectations, beliefs and attitudes, and, of course, what teacher actions might facilitate effective teaching of mathematics? You might think of a mathematics lesson that you have taught that you felt was highly effective. What were the characteristics of this lesson that seemed to make it effective? What decisions did you make and what actions did you undertake that appeared to contribute to the lesson effectiveness? How do you think your knowledge, expectations, beliefs and/or attitudes might have contributed to the effectiveness of the teaching? What role did you give to the children within this lesson? What did you do during the lesson? What sorts of questions did you ask? What was the nature of the mathematics tasks posed to the children and how did they undertake them?

As an example of a response to the question of characteristics of highly effective teachers of numeracy, Figure 1 includes a selection of responses given by a group of final year preservice teachers.

| High student involvement encouraged |
| Enthusiastic                        |
| Use of open-ended tasks             |
| Variety of tasks                   |
| Activities practical or relate to real life |
| Encourages questioning             |
| Utilises student friendly resources |
| Challenges students to think outside the square |
| Displays a positive attitude        |
| Aware of students’ needs and capabilities |
| Focuses on process and not just product |
| Approachable                       |
| Explores how children work out responses |
| Encourages children to explore whether mathematically correct |
| Explains in different ways          |

**Figure 1.** Selection of preservice teacher perceptions of characteristics of highly effective teachers of numeracy.

It is clear that these preservice teachers have developed beliefs about effective teaching of mathematics. The focus is not only on the teacher but interacts also with the role of the child and the nature of the task. It is noted that, from this list, there seems to be less consideration of the teacher role in establishing the mathematical focus within lessons. Perhaps the reader has listed other elements perceived to be characteristic of highly effective teachers of numeracy.
It is good to sit back and reflect on and articulate beliefs as done here. However, it is recommended that these beliefs are then considered in light of findings from research studies of effective teachers of numeracy. Such comparison may extend or refine one’s perceptions of effective teaching of numeracy. While the points listed in Figure 1 may seem a reasonable selection it cannot be assumed what research studies will find.

What does research tell us about highly effective teachers of numeracy?

In a major study of effective primary school mathematics teaching in the United Kingdom, Askew, Brown, Rhodes, Johnson and Wiliam (1997) studied the practices of a number of different teachers, with varying levels of effectiveness. Using mean class gains on a test involving aspects of the number system, computation, and problem solving, aurally administered by teachers with children writing their answers, teachers were grouped into broad categories according to effectiveness. Insights into the practices of effective teachers came primarily from focus schools and case studies of individual teachers, six of whom were identified as highly effective. The highly effective teachers:

• encouraged students to describe their methods and their reasoning, and used these descriptions as a way of developing understanding through establishing and emphasising connections;
• emphasised the importance of using whatever mental, written or electronic methods are most efficient for the problem at hand;
• particularly emphasised the development of mental skills; and
• connected different ideas of mathematics and different representations of each idea by means of a variety of words, symbols and diagrams.

Brown, Askew, Baker, Denvir, and Millett (1998, p. 373) noted that international observational studies ‘seem to show some agreement on some of the aspects of teacher quality which correlate with attainment.’ These included:

• the use of higher order questions, statements and tasks which require thought rather than practice;
• emphasis on establishing, through dialogue, meanings and connections between different mathematical ideas and contexts;
• collaborative problem solving in class and small group settings; and
• more autonomy for students to develop and discuss their own methods and ideas.

Within the Early Numeracy Research Project (ENRP), case studies of six highly effective teachers of numeracy were conducted over a two term period as discussed further below. In addition, a smaller study was conducted of growth in understandings related to Length for children in their first year of school (Sullivan & McDonough, 2002). The study regarding Length revealed marked differences in achievement between classes, irrespective of geographic or socio-economic variables. The research team attributed the differences to the teachers. Self-report data indicated that being effective in the teaching of Length at this level included that the teachers:
• had a clear vision of the mathematical experiences needed;
• were able to engage the students; and
• were prepared to probe the thinking and understanding of the children.

The importance of what the teacher does was noted also by Brown (1999) who stated that ‘quality teaching is more important than class organisation … it’s not whether it’s whole-class, small group or individual teaching but rather what you teach and how you interact mathematically with children that seems to count’ (p. 7).

The six intensive individual case studies within the ENRP provide further insights into the practices of highly effective teachers of numeracy.

### Highly effective teachers of numeracy within the Early Numeracy Research Project

Before discussing the case studies within the Early Numeracy Research Project (ENRP), a brief introduction to the project is provided. The ENRP investigated mathematics teaching and learning in the first three years of schooling, and involved Prep to Grade 2 teachers and children in 35 project (‘trial’) schools and 35 control (‘reference’) schools.

The three key components within the ENRP were the following:
• the development of a research-based framework of growth points in young children’s mathematical learning (in Number, Measurement and Space);
• a 40-minute, one-on-one interview, used by all teachers to assess aspects of the mathematical knowledge of all children at the beginning and end of the school year (February/March and November respectively); and
• extensive professional development at central, regional and school levels, for teachers, coordinators, and principals.

The ENRP framework of growth points in young children’s mathematical learning encompassed nine mathematical domains within three strands: Number (Counting, Place value, Addition and subtraction strategies, Multiplication and division strategies); Measurement (Length, Mass, Time); and Space (Properties of shape, Visualisation and orientation).

Within each domain typically five or six growth points were stated with brief descriptors for each. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children ‘count-all’ to find the total (‘1, 2, 3, …, 11, 12, 13’), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on (‘10, 11, 12, 13’), they can solve the problem in an easier way. Counting All and Counting On are therefore two important growth points in children’s developing understanding of Addition. The ENRP Growth Points informed the creation of assessment items for the one-to-one interview, and the recording, scoring and subsequent analysis.

Identifying highly effective ENRP teachers

While the three key components of the ENRP, as listed above, informed, involved, and potentially empowered the project teachers, it was the teachers, professional learning teams, and schools who ultimately made the decisions of whether and how the information and experiences provided within the project would impact upon their classroom practice. The approach taken fits with Doyle’s (1990) Reflective Professional paradigm according to which both preservice and inservice teacher education should:

foster capacities of observation, analysis, interpretation, and decision making … Within this framework, research and theory do not produce rules or prescriptions for classroom application but rather knowledge of methods of inquiry useful in deliberating about teaching problems and practices. (Doyle, 1990, p. 6)

As described by the ENRP Project Director, Doug Clarke, the approach taken was not to provide teachers with a recipe, but rather with rich ingredients that they might choose in meeting the needs of individual
children, the mathematics and the teaching context. The ENRP teachers were viewed as co-researchers and professionals who would use their own judgment. Thus, the practices of the highly effective teachers were not determined by the researchers and could not be anticipated.

The key criterion for selection of highly effective teachers was student growth. This was ascertained from interview results for 1999 and 2000 showing children’s mathematical growth across the nine ENRP framework domains. The six case study teachers with high student growth were chosen to represent a cross section of grades with one teacher from each of Grade Prep (first year of school in Victoria), Grade Prep/1 Grade 1, Grade 1/2 and Grade 2. One highly effective teacher of Prep children from predominantly non-English speaking backgrounds was selected also for study. The case study teachers had taught within the ENRP and at the same level for the three years of the project, and represented a cross-section of situations such as school location and socio-economic profile. The teachers are later referred to by grade level (e.g. Ms Grade 1), except for the teacher of the non-English speaking background children: Ms NESB. Visits to other ENRP classrooms suggest there were also other highly effective teachers of numeracy in the ENRP but who were not chosen for study because of the limits on number of teachers, grade level, years in the project etcetera.

*Investigating the practices of highly effective teachers of numeracy*

The six teachers were studied intensively through use of the following:

- five lesson observations by teams of two researchers (three consecutive days in the middle of the school year, and two consecutive days a couple of months later), incorporating detailed observer field notes, photographs of lessons and collection of artefacts (e.g. worksheets, student work samples, lesson plans);
- teacher interviews following the lessons (audiotaped and transcribed) to discuss the teacher intentions for the lesson, and what transpired;
- teacher questionnaires completed through the duration of the project; and
- teacher responses to other relevant questions and tasks posed to them.

Many steps were involved in the case study data collection and analysis, beginning with detailed observer notes of each lesson using a laptop to record as much as possible of what was said and what happened, without
interpretation. After each of the lesson observations by two observer/researchers, independent analysis of the lesson was carried out according to a lesson observation and analysis guide made up of nine broad categories agreed upon by the team. These were: Mathematical focus; Features of tasks; Materials, tools and representations; Adaptions/connections/links; Organisational style(s), teaching approaches; Learning community and classroom interaction; Expectations; Reflection; and Assessment methods. Our aim was to describe the practice of demonstrably effective teachers and to look ultimately for common themes, but not to judge.

Following the first three lessons each observer/researcher team produced a summary statement for the teacher they had observed. These statements were shared verbally with the research team at a meeting in which two critical friends, not involved in the research, then provided feedback on the kinds of themes they were hearing. This process occurred again after five lessons had been observed for each teacher.

This observation and analysis process produced a list of themes, which were then cross-referenced again with the data collected to ensure there was evidence of them in action.

The practices of highly effective teachers

From this process a list of 25 practices of effective teachers evolved. It was agreed to list common elements where evidence was available for at least four of the six teachers. These themes are provided in Figure 2 under the nine categories of the ENRP lesson observation and analysis guide, along with the extra category that emerged from the data, that is, ‘personal attributes of the teacher’.
### Effective early numeracy teachers …

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical focus</strong></td>
<td>• focus on important mathematical ideas&lt;br&gt;• make the mathematical focus clear to the children</td>
</tr>
<tr>
<td><strong>Features of tasks</strong></td>
<td>• structure purposeful tasks that enable different possibilities, strategies and products to emerge&lt;br&gt;• choose tasks that engage children and maintain involvement</td>
</tr>
<tr>
<td><strong>Materials, tools and representations</strong></td>
<td>• use a range of materials/representations/contexts for the same concept</td>
</tr>
<tr>
<td><strong>Adaptions/connections/links</strong></td>
<td>• use teachable moments as they occur&lt;br&gt;• make connections to mathematical ideas from previous lessons or experiences</td>
</tr>
<tr>
<td><strong>Organisational style(s), teaching approaches</strong></td>
<td>• engage and focus children’s mathematical thinking through an introductory, whole group activity&lt;br&gt;• choose from a variety of individual and group structures and teacher roles within the major part of the lesson</td>
</tr>
<tr>
<td><strong>Learning community and classroom interaction</strong></td>
<td>• use a range of question types to probe and challenge children’s thinking and reasoning&lt;br&gt;• hold back from telling children everything&lt;br&gt;• encourage children to explain their mathematical thinking/ideas&lt;br&gt;• encourage children to listen and evaluate others’ mathematical thinking/ideas, and help with methods and understanding&lt;br&gt;• listen attentively to individual children&lt;br&gt;• build on children’s mathematical ideas and strategies</td>
</tr>
<tr>
<td><strong>Expectations</strong></td>
<td>• have high but realistic mathematical expectations of all children&lt;br&gt;• promote and value effort, persistence and concentration</td>
</tr>
<tr>
<td><strong>Reflection</strong></td>
<td>• draw out key mathematical ideas during and/or towards the end of the lesson&lt;br&gt;• after the lesson, reflect on children’s responses and learning, together with activities and lesson content</td>
</tr>
<tr>
<td><strong>Assessment methods</strong></td>
<td>• collect data by observation and/or listening to children, taking notes as appropriate&lt;br&gt;• use a variety of assessment methods&lt;br&gt;• modify planning as a result of assessment</td>
</tr>
<tr>
<td><strong>Personal attributes of the teacher</strong></td>
<td>• believe that mathematics learning can and should be enjoyable&lt;br&gt;• are confident in their own knowledge of mathematics at the level they are teaching&lt;br&gt;• show pride and pleasure in individuals’ success</td>
</tr>
</tbody>
</table>

**Figure 2.** Common themes emerging from six individual ENRP case studies.

A classroom vignette, teacher interview statements and observer/researcher summaries are shared below to illustrate practices of these teachers.
Illustrating effective practices: A sample lesson

To provide some illustration of practices of effective teachers, one lesson taught by Ms Prep is described below. It should be noted that no two of the six highly effective teachers taught in exactly the same way, and that any one of the 25 characteristics might be enacted in different ways.

While reading this lesson outline you may wish to reflect upon which characteristics of effective teaching as identified in the ENRP (Figure 2) you believe are evident here.

In this lesson, Ms Prep wanted to “introduce the children to repeating patterns … They had noticed numbers on the number chart but we hadn’t done anything with other patterns at all”. Ms Prep introduced the lesson through the reading of a short book called Patterns that included reference to patterns in leaves, sand, and wings of butterflies. She then brought out a bag of ‘shopping’ containing gifts for family members who were having birthdays that month. The first item she took from the bag was a Telly Tubbies plate that attracted the interest of these five to six year olds! One child noticed there was a pattern on the plate. The teacher asked, “How many colours are in the pattern?” She also stated, “Around the plate is a repeating pattern”.

In turn, a patterned handtowel, and patterned sheets of wrapping paper were shown to the children. The teacher asked questions such as: “See if there is a pattern on my wrapping paper”. “[Child A] could you tell me if you can see a pattern? What is the pattern?” Child A volunteered that he saw another pattern. The teacher asked him to describe it. The teacher then asked other children, “What pattern do you see?” “[Child B] can you see any pattern there? Is there a repeating pattern?” A number of patterns were identified and described. The teacher emphasised the nature of patterns, that is, the repetition.

About 15 minutes had passed and the children now sat in a circle on the floor. One child was given a container of square, plastic tiles of various colours. The teacher asked, “[Child C] see if you can make a repeating pattern and we’ll see if we can work out what it is.” Once the child had made a pattern the teacher asked her: “[Child C] could you tell us your pattern please?” The teacher then addressed the class, “If that is a pattern, what would be the next colour you put in?” She praised the child for her pattern – “That was a very hard pattern [Child C], you were very good at that”.

Andrea McDonough
The teacher then made a pattern – Green, red, green, red, and asked, “[Child D] what’s my pattern? What would go next?” The teacher then informed the class, “I am going to give you some containers. I would like you all to make your own pattern. Try and make it a repeating pattern. When you finish we will look at your pattern”. The children made a variety of patterns (e.g. Figure 3).

Figure 3. Making patterns.

Rather than commenting herself on the children’s patterns, the teacher gave this role to the class by saying things such as, “[Child E] tell us your pattern. Is that a repeating pattern? Let’s all say [Child E’s] pattern. How many colours make up [Child E’s] pattern?” She did the same for another child – “[Child F] tell us your pattern. Is that a repeating pattern?” She then asked another child to comment on the pattern made – “[Child G] why is it not a repeating pattern? Where did it stop being the same? How can we make it a repeating pattern?” For another she said, “How could he make that a repeating pattern? Anybody got any suggestions?”

Another 15 minutes had passed. The lesson then continued with the same mathematical focus but with different materials. Within the context of preparing for a party that was to occur the following day, the teacher introduced the idea of decorating a clear plastic cup. After demonstrating how to stick coloured stars on the cup she stated, “I’d like you to use your imagination and try and make a repeating pattern around the top of the cup”. The children were free to choose the number of colours in their pattern, thus there was some openness in the task. While each child decorated a cup, the teacher assessed whether the children were able to make a repeating pattern. The teacher roved and recorded this on a piece of paper.

Following this task the children moved to the floor and the class discussed some of the patterns. The teacher read them out and the children were asked
to tell her whether each was a repeating pattern. She asked questions such as, “Is that a repeating pattern? Does it do the same thing over and over again?” Some children were observed to self-correct, such as one child who stated, “I forgot a red”.

REFLECTING UPON THIS LESSON

This lesson contained many elements that suggest the teacher possessed and implemented a range of characteristics of effective teachers of numeracy. Just some of these are discussed below.

One element that stands out is a continuing focus on the important mathematical idea of pattern. The idea of repetition within pattern was reinforced through the use of a range of materials/representations/contexts for the same concept. The teacher believed that the use of the book and the bag of shopping allowed children to “relate patterns to elsewhere, patterns in the environment” and thus the teacher made connections to mathematical ideas from children’s experiences.

The mathematical focus was made clear to the children, for example, through questioning the children about whether they and others had made repeating patterns, and by stating the idea of a ‘repeating pattern’ a number of times, as well as by giving the children the opportunity to examine and make a range of varying patterns.

The teacher also structured purposeful tasks that enabled different possibilities, strategies and products to emerge, that is, the tasks had some openness. A range of patterns was possible when using both the counters and the stars. Each pattern might have had anything from two to about five or six repeating elements. There were also a number of tasks and changes of activity within the lesson. The teacher explained, “I have to. Preps lose interest if you dwell on one aspect too long. You have to have short sharp activities with Prep”. Nonetheless, each task was focused on the mathematical idea and through the use of a range of materials allowed the children to draw out the common element, that is, the mathematics.

Many characteristics within the Organisational style(s), teaching approaches, and Learning community and classroom interaction categories (Figure 2) were evident in this lesson. For example, the children were encouraged to explain their mathematical thinking/ideas and to evaluate others’ mathematical thinking/ideas, and help with methods and understanding.
The teacher was not the only person in the class responsible for reflecting upon whether the children had met the requirements. In the interview the teacher stated:

I don’t think you can just say that’s wrong … I prefer the other children to say no, that’s not a repeating pattern than for me to say it. I don’t like to be the bad guy perhaps, but I think it’s good when the other children can recognise it’s not. And I can say, well how can we make it one. We all pitch in their ideas to make it.

She also stated, “If I say, how did you work that out, it just reinforces the strategy that they’ve used”. However, she also believed there were times when she had to actively intervene:

I like them to be involved with what we’re doing. And as much as possible I like them to try and work it out or see it before I actually tell them. But sometimes you just have to tell them. Sometimes you do. Some children just don’t pick it and if you tell them, they think, Oh yeah! Perhaps that will give them that little start and then they’ll start to think things. I do like to get them involved and I do try to let them point things out as much as I can before I’ll show them … as if they’ve discovered it and they… I’ll look, Oh yes, well done, well let’s have a look at that, if they’re leading where we’re going perhaps.

Ms Prep also seemed to listen attentively to individual children when interacting with them. These discussions had the characteristics of an individual conversation, even when student initiated. The summary report following the first three visits included the following comment:

[Ms Prep] was very positive towards the children. She praised them for their thinking. She showed caring and respect for individual children, giving them her attention even within a group situation. She answered individual children’s questions and followed their train of thinking, unless it was totally off the topic.

Another feature of this lesson was that Ms Prep drew out key mathematical ideas during and/or towards the end of the lesson. While there was a share time and drawing together towards the end of the lesson, this reflection also occurred through the lesson.
Another of the 25 characteristics evident within this lesson was the *collection of data by observation and/or listening to children, taking notes as appropriate*. Being clear on the mathematical focus of the lesson, Ms Prep was able to do this meaningfully.

**A comment on the ENRP findings**

The intention of presenting the detail of Ms Prep’s lesson was not to outline a perfect lesson! However, it is a lesson that illustrates many of the 25 characteristics of effective teachers identified within the ENRP individual case studies. Even for this teacher these characteristics became evident in different ways in other lessons observed. No two lessons looked the same just as the teaching of no two of the highly effective teachers studied within the case studies looked the same.

One feature of these teachers was their ability to integrate a number of these features within any one lesson and make it look effortless. However, there was evidence that these teachers reflected upon what they knew of their children’s mathematical understandings, upon the mathematics that the children were moving towards, and upon the effectiveness of their teaching strategies. In turn they planned their mathematics classes with care. Having undertaken this reflection and planning they could make effective fine grain decisions as they were teaching their numeracy lessons. As identified by Boaler (2003), it is not only the features apparent in teaching that are important, but it is how they are enacted that is important. It is clear that teachers should consider not only what they do but *how* they do it.

**Conclusion**

Teaching is a complex craft involving many skills. It can also involve the artistry and enjoyment of combining these skills into cohesive and effective lessons that are productive learning opportunities for children. As identified in the opening quote from Hargreaves (1994), teachers each play a key role in the learning experiences of the children they teach.

The purpose of this paper was to consider characteristics of highly effective teachers of numeracy. The listing of characteristics resulting from the ENRP and the identification of features from other research provide the opportunity for the reader to reflect upon, and perhaps extend or refine, his or her list of perceived characteristics of effective teaching.
of numeracy. The findings from the ENRP case studies have challenged me to examine what I value in mathematics teaching. Since having the privilege of observing many lessons taught by ENRP teachers, and most particularly 15 lessons within the classrooms of three of the ENRP highly effective teachers of numeracy, and analysing and reflecting upon what makes the six teachers highly effective, I have found myself implementing some of these characteristics, sometimes consciously and sometimes unconsciously, in both primary and tertiary settings. It is my belief that the characteristics of highly effective teachers of numeracy, as outlined in this paper, can apply to teaching in the early years and beyond.

NOTES

1 The Early Numeracy Research Project (ENRP) was a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project was funded in 35 project (‘trial’) schools and 35 control (‘reference’) schools. The views presented here are those of the author.

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