Using Graphics Calculators with Low-Achieving Students

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This paper reports on a yearlong graphics calculator-intensive teaching experiment that was conducted with two low-achieving senior mathematics students. Whilst constructivist approaches to mathematics teaching are being actively promoted to improve student learning outcomes, the author found that a carefully balanced mix of direct and constructivist approaches is more suitable for this cohort of students. A simple, easy-to-use tool-based scoring rubric is also presented that helps teachers measure, at a glance, the performance of their students.

Introduction
As mathematics teachers we are constantly interacting with students and the purpose of this interaction is to ‘get inside’ their collective minds so that we can make sense of their mathematical realities. As challenging as that may be I would argue that the task facing teachers and, more specifically, the low-achieving members of your class is even more testing. And just to compound the issue further we are confronted with another twentieth century phenomenon, the graphics calculator.

Over the past two decades mathematics curricula, particularly at senior level, have incorporated innovative technologies at an ever-increasing rate (Lindsay, 1999). Underpinning these initiatives is a basic assumption: that technology per se has the potential to better assist all students with their
This article questions the validity of such an assumption when applied to a group of low-achieving senior students using graphics calculators, and suggests how we can use simple strategies to assist the weaker members of our classes.

**Barry**

Consider, for example, Barry, aged 18, who had just recently embarked on a university course for which he was poorly prepared. Entry level testing indicated that both his algebraic and graphing skills were of a ‘bare’ Year 10 standard (Lindsay, 2002a). He had used a graphics calculator at secondary school. However, as is common with low-achievers, just *how* he used it was the issue here. For instance, the following question concerns a girl throwing a stone out to sea from the top of a vertical cliff. The path of the stone is given by 

\[ y = 40 + 2x - \frac{x^2}{10} \]

where \( y \) is the height of the stone above sea level and \( x \) is the horizontal distance of the stone from the cliff. I discussed this question with Barry in one of three one-hour interviews that took place during the course of the year. The graphics calculator used in this study was the TI-83. His interpretation of the problem together with its correct representation are shown in Figure 1.

![Figure 1](image)  

*Figure 1.* Barry’s sketch of the ‘cliff’ question (left), and its correctly scaled representation (right).

An extract from the interview is included below (I: Interviewer; B: Barry; brackets denote interviewer’s comments):

I. … right, it (ball) goes up to 40, and then what happens to the stone?  
B. It drops down, that is the highest height it reaches.
I. That’s the highest point at 40 (interviewer pointing to the Y-intercept)?
B. Yeah.
I. How does that agree or differ with what you said about the height of the cliff?
B. Mmm …
I. Right, so which one (equation or graph) are you going for now?
B. The graph.
I. Okay, so how high does the stone go?
B. 40.
I. … and how far from the foot of the cliff does the stone strike the water?
B. Well, from the cliff …
I. Where are you calling the foot of the cliff?
B. Isn’t that here (pointing to the negative X-intercept – see sketch above – where Barry has written -9)
I. -9?
B. No … I am guessing …32 (Barry indicates the positive X-intercept).
I. … so again, can you just go over what you understand by that graph that you have drawn, and link it to the girl and the cliff and the stone.
B. Just like from the equation … I just pretty much guessed it was 40, but then … the maximum height … 40, so that is the maximum height is … I don’t know if these are really … I am not sure …

It was clear from the discussion that Barry had struggled with the problem. Through weekly classroom discussions, and follow-up interviews, I was able to get to know Barry’s mathematical thinking fairly well. It was evident that he was using the tool as a ‘button-pushing’ device. It is instructive to note the absence of detail in Barry’s sketches; graphs were copied from the calculator screen with no evidence to suggest that he was making the necessary connections between his sketch and the problem at hand; scales were omitted from axes, and values such as axial intercepts were generally missing. There was also little or no evidence to suggest that he was correctly using mathematical language, and his connections between graphical and symbolic representations were tenuous to say the least.
I called students’ responses, of which Barry’s are typical, computational responses. Computational users are characterised by the following indicators of performance (Lindsay, 2002a):

- Output is being copied regardless of its accuracy or relevance
- Limited or no progress towards completion of the task, or no response to the task
- Incorrect and inaccurate use of mathematics
- Mathematics explanations brief or absent; limited technical ability in using the graphics calculator
- Employs graphics calculator-generated solutions instead of algebraically derived solutions
- No evidence of correct use of mathematics language, symbols and conventions
- No links between graphical, symbolic and numerical representations
- Makes no attempt at moving to generalisations.

Teachers probably have no trouble in identifying with many of these descriptors with the weaker members of their class. And while it may be unfashionable to say so, constructivist strategies were found to be not only unhelpful in assisting Barry, they were considered counterproductive as well. In fact, constructivist approaches had provided the focus for the pedagogical orientation early on in my study with Barry and his classmates (Lindsay, 2002b) – each lesson began with group work whereby students were encouraged to share their experiences with each other, my role being one of facilitating rather than leading. However, it soon became apparent that such an approach did not suit the learning styles of this particular cohort of students. To many, discovery-based approaches to learning were too confusing and unstructured, and my students’ ability to ‘make sense’ of their mathematics was hampered time and time again by a lack of background pencil-and-paper skills and knowledge. So, while constructivism as an epistemological belief – that we can only construct personal models of the knowledge and experience of others (Goldin, 1990) – is and will continue to be the philosophy of my teaching, a constructivist pedagogy was deemed inappropriate in this particular situation. Thus, typical lessons were structured so that teacher-centred instruction of pencil-and-paper skills was interspersed with plenty of practice to consolidate what students had learned. This was then followed by more unstructured user-centred,
technology-based activities which included problem-solving and applications-type tasks that were directly related to the skills previously learned and, more broadly, to students’ experiences.

The task of assisting Barry consisted of providing him with a skills and knowledge base from which he could engage meaningfully with the graphics calculator. Only by continually moving back-and-forth – or ‘folding back’ (Pirie & Kieren, 1994) – between his newly acquired pencil-and-paper (P/P) skills and his tool-based tasks (shown in Figure 2 below by a circular broken line) was he able to achieve tool-related success at the most basic level. Once mastery had been achieved at the computational stage he was then able to progress on to more cognitively demanding activities.

![Figure 2. The limited learning pathway of Barry, a computational user.](image)

**Anna**

Anna, aged 17, was another member of my class. Like Barry she had used a graphics calculator before, and entry-level testing revealed that she was operating at Year 11 standard with respect to her algebraic and graphing skills. What interested me about Anna, however, was her unusually high level of enthusiasm for and expertise with the graphics calculator, and it was this aspect of Anna’s mathematics that I will describe here.

In the second of three interviews that were conducted with her during the year, I began by asking Anna to recount a question that she had answered in her end-of-semester 1 examination paper. It required knowledge of the function, \( y = x^3 - 12x^2 - 5x + 6 \). The default graph-plot reveals only part of the function, the user needing to ‘zoom’ out twice and change the scale in the ‘window’ menu to identify the full graph. Anna correctly zoomed out and was able to capture the function’s global behaviour with relative ease. Her graph is shown in Figure 3.

Anna was asked to explain how she worked through this question. The following discussion took place (I: interviewer; A: Anna; brackets denote interviewer’s comments).
I. Type in \(x^3 - 12x^2 - 5x + 6\) and tell me what you’re doing.
A. Well, you can tell that the X-axis is being crossed a couple of times
because it is the \(x^3\) … I have just seen it (x-intercept) there, and it
finishes up …
I. You are pointing to the right of the screen?
A. Yeah, and ‘zooming’ out again, hitting the ‘window’ key … and see
where it all touches the x-axis, so it goes there and I just hit ‘trace’
and … to the x-axis … just keep ‘tracing’ … now here we go, we
have got all the intercepts ….
I. You have got the three intercepts on the x-axis?
A. Yeah, and then you just ‘trace’ that to get up closer so that you know
one of them is 12, one is .5, the other one is -.9, so … yeah.
I. Okay, and the y-intercept would be?
A. Where it … so ‘zoom’ out again … where it touches the y-axis …
I just keep ‘zooming’ around and play with it a bit.
I. … until you have got the y-intercept?
A. Yeah, until you get that … and a nice view (the entire graph is now
in view).

Figure 3. Anna’s graphics calculator sketch of a cubic function.

Anna’s knowledge of cubic functions (“the x-axis is being crossed a couple
of times”), combined with an improved technical proficiency in using the
graphics calculator enabled her to successfully answer this question. She
was prepared to “play with it (graphics calculator) a bit” and this, it is suggested,
contributed to a successful solution.
It is most likely that there are students like Anna in your class who are sufficiently motivated and proficient in accomplishing tool-based tasks. I called students’ responses, of which Anna’s are typical, technicians’ responses. Technicians are characterised by the following indicators of performance:

- Makes some progress towards completing the task
- Makes some progress towards correctly and accurately using mathematics
- Limited, though purposeful explanations and interpretations
- Is usually technically competent in using a graphics calculator
- Uses graphics calculator-generated solutions instead of algebraically derived solutions
- Some mathematics language, symbols and conventions used correctly
- Partial but limited links between graphical, symbolic and numerical representations
- Unable to move to generalisations

Technicians have greater control of and familiarity with the common graphics calculator routines. Unlike computational users, any deficiencies in technicians’ pencil and paper skills are overcome by a willingness on their part to use the tool to help them (“to play with it a bit”) with their mathematics. Technicians also have a less static conception of screen images, a feature that characterises computational users. For instance, the default images (the ‘Zstandard’ viewing window) on the graphics calculator screen confronting Barry were regarded as the correct images. He was reluctant to consider that such images might be incorrect, and that ‘scaling’ was required to view a function’s global properties. Moving from a static to a dynamic conception of screen images requires an ability to consider the properties of these images. That is, students must be able to link images with an understanding of their mathematics, and this is a characteristic feature of multi-representational thinking, episodes of which were apparent in Anna’s solution responses.

Technicians are more expert at interpreting visual images, and they are beginning to demonstrate partial links between symbolic, graphical and numerical representations (see the broken-lined unidirectional arrow in Figure 4). However, regardless of the level at which she was operating, Anna always ‘folded back’ (Pirie & Kieren, 1994) at appropriate times to retrieve and
reorganise her mathematical ‘infrastructure’ (shown in Figure 4 by circular broken arrows). Integration of information was largely successful, with screen images such as graphs being correctly adjusted (‘zoomed’, ‘traced’, etc) so that their global behaviours could be captured.

Figure 4. The learning pathway taken by Anna, a technician. Occasional episodes of multi-representational thinking were also evident.

The nature and extent to which Barry and Anna were able to progress to higher levels of tool-based engagement and performance was shaped by the interaction of three components (see Figure 5).

Figure 5. The interactions between students’ prior skills and knowledge, students’ attitudes and pedagogy that mediate students’ subsequent tool-based performances.
Students’ background skills and knowledge

For instance, Barry’s task responses were characterised by an incomplete repertoire of pencil-and-paper skills and knowledge that restricted his ability to explain tool-based images. Consequently he was observed to engage in ‘button pushing’ routines, and screen outputs were copied regardless of their accuracy or relevance. Whilst the technology was capable of providing the necessary scaffolding, his progress was dependant on him possessing the required mathematical ‘infrastructure’

Students’ attitudes

For instance, Anna’s willingness to embrace the graphics calculator and her subsequent proficiency in using it is believed to have assisted her in making significant progress with tool-based tasks

Classroom pedagogy

For instance, lessons were structured so that teacher-centred instruction of pencil-and-paper skills was interspersed with plenty of practice to consolidate what students had learned. This was then followed by more unstructured user-centred, technology-based activities which included problem-solving and applications-type tasks that were directly related to the skills previously learned.

The graphics calculator has the potential to effect real change in the classroom and alter the type of mathematics that is learnt and taught. However, to effect such change requires the recognition that within your classroom there are students with a diverse range of backgrounds and needs, and that these individuals must be accommodated in different ways through the appropriate use of instructional activities.

References


