The challenges of the teaching and learning of mathematics for teachers and students respectively in the middle years of schooling have been well documented by a range of recent Australian and overseas studies. Following a brief discussion of these, a range of strategies, classroom activities and assessment tasks is presented which may offer an appropriate way forward, and increase the confidence and capabilities of mathematical thinkers in these important years.

Mathematics teaching and learning in the middle years: So many challenges

The Executive Summary of Beyond the Middle (Luke et al., 2003), a report commissioned by the Commonwealth Department of Education, Science and Training into middle schooling in Australia, included the following statements:

• There is a dominance of literacy as a policy priority across all sectors. The focus on literacy appears to have occurred at the expense of numeracy.
• The assessment procedures used in middle years varied greatly and were not overall of high standard.
• There needs to be a more systematic emphasis on intellectual demand and student engagement in mainstream pedagogy. … This will require
a much stronger emphasis on quality and diversity of pedagogy, on the spread of mainstreaming of approaches to teaching and learning that stress higher order thinking and critical literacy, greater depth of knowledge and understanding and increases in overall intellectual demand and expectations of middle years students. (pp. 5–8)

In Victoria, the Middle Years Numeracy Research Project (Siemon, Virgona, & Corneille, 2001) collected baseline numeracy performance data from a structured sample of Year 5 to 9 students from 47 schools. Twenty ‘trial schools’ were selected to explore ‘what works’ in relation to improving numeracy outcomes. Among the findings and recommendations were the following:

- Teachers in the middle years can and should expect a range of up to seven school years in numeracy-related performance.
- There is a significant ‘dip’ in student numeracy performance from Year 6 to Year 7 [the transition years from primary to secondary in Victoria].
- *Opportunity to learn* is as much a factor in explaining differences in performances as so-called *ability*.
- Fractions, decimals, multiplicative thinking and the capacity to interpret, apply and communicate what was known in context were among the most common sources of student difficulty. (pp. 5–7)

I see teacher knowledge, both content knowledge and *pedagogical content knowledge* (Shulman, 1987) as an underpinning problem of many of these issues. Teacher knowledge is crucial, and contributes greatly to many of the issues and challenges discussed above (Ma, 1999). For, as Brophy (1991) argued:

where teachers’ knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond fully to student comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasise interactive discourse in favour of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352)

So, there are many challenges facing teachers, schools and systems in improving both cognitive and affective aspects of students’ mathematics learning in the middle years. In the remainder of this paper, I will offer some
personal strategies, with appropriate examples, that may contribute to a more positive experience for the students in these important years, both cognitively and affectively.

**Some strategies, approaches and activities with potential for enhancing mathematics learning in the middle years**

*Valuing and building upon students’ methods of solving problems*

Many readers will recall that when they were at school it seemed that there was one way to solve a given problem in mathematics – either the teacher’s way or the textbook’s way. And if you weren’t able to use the approved method, you were highly disadvantaged in a range of ways, including during assessment. Increasingly, reports from noted scholars in Australia and beyond (see, e.g. Kilpatrick, Swafford, & Findell, 2001) are advocating a far greater emphasis on looking at particular topics and particular problems in far more depth, and valuing a variety of methods in solving these problems. Statements such as the mathematics curriculum being ‘a mile wide and an inch thick’ (often heard in the U.S. context), and the belief that ‘less is more’ are increasingly common.

The reader is invited to take a moment to solve the following problem in their head: What number is half-way between 39 and 103?

For most middle school students who are successful, the following methods are common:

- Using the conventional written algorithm mentally to subtract 39 from 103, with trading along the way. The answer is then halved and added to 39, giving 71 as the answer. Of course using the conventional written algorithm for a mental task is very challenging, as there is much to be kept in the head when using this method.
- Changing the numbers in some way to make the problem less ‘messy’, such as working with 40 and 100, and then making suitable adjustments.
- Adding the two numbers together, and halving the total, thereby finding the average.
- Zooming in on the middle number through progressive stages, usually relying upon a visual image of a number line or something similar. (i.e., the problem is the same as half way between 49 and 93, or 59 and 83, or 69 and 73, the final pair being much easier to work with.

When I use this problem with teachers and students, my point isn’t to emphasise that one method is better than the others, but rather that in a classroom where
such methods are sought and valued, students may feel encouraged greatly to
know that their method is ‘okay’. Of course, part of the classroom dialogue
can focus on the strengths and weaknesses of the various methods, and the types
of problems for which certain methods would work best, without giving
students a feeling of rejection. Anecdotally, I can report many mature-age students
at my university commenting that “for the first time, my methods are valued”,
and the consequent feeling of empowerment that results.

This kind of mental arithmetic, where strategy is more important than
speed and competition traditionally associated with mental arithmetic in
school classrooms, is increasingly important in light of the data from
Northcote and McIntosh (1999) that showed that 200 adults on average used
mental strategies for more than 84% of calculations. They also noted that
around 60% of calculations required only an estimate, giving greater
impetus to the need to emphasise estimation in classroom activity.

An interesting final note in this section is the advice of George Polya, the
‘grandfather of mathematical problem solving’ that “it’s better to solve one
problem five different ways than five different problems the same way” (source
unknown). It is interesting that the TIMSS video data showed that Japanese
primary school teachers were very likely to make a single problem the focus
of classroom work on a given day. Japan is regularly one of the most
successful countries in mathematics in international comparative studies.

**Asking higher order questions**

Schoenfeld (1992) was concerned that high school students were not
sufficiently metacognitive in their approach to solving mathematics
problems. He claimed that roughly 60% of all students’ problem solving
attempts (as recorded on video) could be characterised as ‘read the problem,
make a decision quickly, and pursue that direction come hell or high water’
(p. 356), very different from the process that mathematicians used in
solving problems during his study. He wanted his students to become more
like the mathematicians, with a suitable level of self-regulation, monitoring
and control. In his classroom, approximately one-third of class time was
spent on the students working on problems in small groups, while he roved
around the classroom. As a strategy to increase metacognition, he told his
students that as they worked away in groups, he reserved the right, at any
time, to ask the following three questions:
• What (exactly) are you doing? (Can you describe it precisely?)
• Why are you doing it? (How does it fit into the solution?)
• How does it help you? (What will you do with the outcome when you obtain it?)

Interestingly, as the students started to become familiar with this routine, they began to defend themselves by discussing the answers to these questions in advance. By the end of the semester, they had so internalised this approach that Schoenfeld found that he no longer needed to ask these questions, and when he videotaped the students during further problem solving attempts, he noticed that their behaviour was much more like the mathematicians than previously.

As part of the Early Numeracy Research Project (see, e.g., Clarke, 2001), 353 teachers in 35 schools used a one-to-one, assessment interview with children in Grades Prep to 4, for about forty minutes per child, at the beginning and end of the school year. The interview was very interactive, with much emphasis on students explaining their thinking. It was interesting to note that the interview provided a kind of model of the questions that teachers could ask their children in everyday interactions in mathematics. Teachers reported that the following kinds of questions were increasingly common by the third and final year of the project (Clarke, et al., 2002):

• What happens if I change this here …?
• What could you do next?
• Is there a pattern in your results?
• Tell me about your pattern.
• Is there a quicker way to do that?
• How are these two problems the same and how are they different?
• Can you make up a new task using the same materials?
• How could you make the problem more difficult?

The argument is that the kinds of questions above in the context of students working on rich and worthwhile mathematical problems will promote the kind of higher-order thinking advocated by so many.

Offering students choices and openness in mathematics activities and assessment

When I was piloting tasks for two projects in which I was involved in developing ‘rich’ assessment tasks (see, Beesey, Clarke, Clarke, Stephens, & Sullivan,
I was often in the position of wanting to trial a number of tasks within the one lesson. As a consequence, I would put a pile of tasks at the front of the room, and invite students to come and choose one to work on that was of interest to them.

This process gave me an opportunity to see the bases for their decisions, but also showed me how much the students appreciated being given a choice. It occurred to me at the time that the simple strategy of trying to increase the opportunities for students to make a choice during mathematics lessons and during formal assessment times would be well received by students.

A colleague in the United Kingdom showed me the way in which he and his collaborators were using ‘Can do’ assessment tasks, which also had this element of choice. As teachers were accountable for students being able to demonstrate certain outcomes, they decided to phrase assessment tasks accordingly, using the words of the outcome statements, with an invitation for students to show that they can do the outcome, in a way that the student considers appropriate. In this way, the students were given a choice about how they were going to demonstrate that they could achieve that outcome.

Another related approach is the use of open questions. These potentially have the advantage of enabling students to respond in a variety of ways, according to their level of understanding. One of the simplest yet powerful examples of this is the following task, developed by Peter Sullivan (Figure 1).

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**Figure 1.**

What do you think this might be the graph of?

Put names and numbers on the graph to show what you mean.

Write down three things which you know from your graph.
Students appreciated the opportunity to choose the context for this problem. Barbara Clarke and I used it with over 100 Grade 5 students. Although 12 students used the context of ‘favourite sports’ or ‘sports I play’ and 6 used ‘favourite pets’, every other context chosen by the students was unique. The students appreciated the opportunity to determine the context. The teachers found the student work wonderfully revealing of their understanding of labeling graphs, scales, and interpretation. Contrast this with the classic textbook problem, where the students would be told, for example, that this graph represented how many of each type of pizza were sold at a store on a particular day, and then asked various (trivial) questions about these data. This task provides much more information for the teacher, and is of far more interest to the students.

Making hard decisions about less important content

It seems amazing in 2003 that this point needs to be made, but it is still the case that much of the content that has been recognised in so many countries as inappropriate for major emphasis in the middle years still remains the staple diet of many middle school students in Australia. Given all of the new content that is needed to meet the demands of an increasingly technological society, we can no longer justify giving substantial classroom time to topics such as multi-digit long division and complicated manipulation of fractions and decimals.

As mathematician Professor Zal Usiskin (1984) put it nearly twenty years ago, “too much else should be learned about mathematics to waste time practicing obsolete skills. Mathematics is getting easier. We will not be able to keep this from our students forever” (p. 264).

Building in an element of challenge and excitement to enrich traditional content

There is a variety of ways of making middle school mathematics more enjoyable and challenging. One way is to take traditional content and build in a problem solving component to it. The following task is one I adapted from Middleton & Goepfert (1996). It is explained to students that if we placed a normal measuring tape around a tree we would be measuring the circumference. However, in the timber industry, they often want a quick way to measure the diameter, to decide whether a given tree is ready to be felled.
They have developed a commercial instrument for this purpose, called the ‘D-Tape’. This is placed around the tree, approximately 2 metres from the ground, and is calibrated to read off the diameter directly. Students are challenged to work in groups with paper streamers, calculators and rulers, to create their own D-Tape. An exciting, culminating highlight of this activity is to have every group come out and measure a sample ‘tree’ which could be a large rubbish bin or a giant pot for a plant. The group estimate which is closest to the teacher’s pre-measured diameter is the winner.

**Recognising the effects of ability grouping in mathematics**

A common discussion point in education is the appropriateness or otherwise of ‘ability grouping’ in general, and in mathematics in particular. Sometimes it is hard to separate our personal philosophies from what research says. An interesting meta-analysis was conducted by Lou, Abrami, Spence, Poulson, Chambers, & d’Apollonia (1996). Using 165 studies, across a range of grade levels and curriculum areas, they drew conclusions about the effect of small group work and ways of organizing the classroom. Two conclusions are of interest here:

- Students working in small groups achieved significantly more than students not learning in small groups.
- The subject area made a difference: There were no significant differences between ability grouping and mixed ability grouping in mathematics, compared with significant differences in reading in favour of homogeneous groups.

In the light of this research, I think that teachers need to think carefully about their reasons for choosing to place students into groups according to perceived ability. If the research is showing no significant differences, then we need to consider the potential impact upon students’ self-esteem, and also the potential for what Brophy (1963) calls ‘the self-fulfilling prophecy’, where students perform to the level expected of them by their teacher. My observations over many years and conversations with teachers have led me to conclude that ability grouping is used in mathematics for teacher convenience more than for student benefit. Frequently, advocates claim that movement between groups is frequent, but to my mind, it is rarely evident. Another issue is that many teachers believe that by ability grouping, they have removed largely the differences between students, leading to the
possibility of a teacher teaching to the middle, or as a Peanuts cartoon once put it, aiming for the middle pin when bowling, and hoping to hit as many others along the way.

**Developing and using rich assessment tasks**

No subject is so associated with its form of assessment as is mathematics. At the same time, Clarke (1989) argues that it is through our assessment that we communicate most clearly to students those learning outcomes we value.

If the University of Queensland study reported earlier is representative of what is happening in our classrooms, then our assessment needs as much consideration for renewal as does content and classroom pedagogy. A helpful message to teachers is possibly ‘not more assessment, but more appropriate assessment.’ Many would argue that our students are currently over-assessed in middle school classrooms, and much of the data collected is never used. A note purportedly on Albert Einstein’s wall was possibly his contribution to the assessment debate: “Not everything that counts can be counted and not everything that can be counted counts.”

In conversations with teachers, I have assembled gradually a list of desirable characteristics of assessment tasks that could reasonably be described as ‘rich’. The list is as follows:

*Rich assessment tasks:*

- connect naturally with what has been taught;
- address a range of outcomes in the one task;
- are time efficient and manageable;
- allow all students to make a start;
- engage the learner;
- can be successfully undertaken using a range of methods or approaches;
- provide a measure of choice or ‘openness’;
- encourage students to disclose their own understanding of what they have learned;
- allow students to show connections they are able to make between the concepts they have learned;
- are themselves worthwhile activities for students’ learning;
- provide a range of student responses, including a chance for students to show all that they know about the relevant content;
• draw the attention of teachers and students to important aspects of mathematical activity; and

• help teachers to decide what specific help students may require in the relevant content areas.

Of course, few tasks would satisfy all of these criteria, but by having this as a kind of checklist, the quality of our assessment tasks is likely to increase.

I am asked frequently how this list differs from a list of the features of rich classroom activities. This is a most reasonable question. In my mind, the major difference is the purpose of the activity. In the case of an assessment task, the teacher has a particular mathematical focus in mind, about which they want to collect information about individuals or the group. Of course, it remains the case that the richest form of assessment information that the teacher has access to is the data they collect through the things they see and hear during day-to-day classroom interactions with individuals, small groups and the whole groups (Clarke & Wilson, 1994).

Although many examples of rich assessment tasks could be given, only one is presented here, *Helping Bert Do Division*. In this task, middle school students are presented with three division problems that ‘Bert’ has worked on. As becomes evident, he is having difficulty using 0 as a place-holder when dividing. Students are encouraged to consider Bert’s work for correctness, offer some advice to Bert, write down one question that Bert would get correct, and then three that they predict Bert would get incorrect, and give both the correct answers and Bert’s probable answers. One student’s work, ‘Wade’, is presented below. Although Wade has made a small slip, his work reveals an excellent understanding of the division process (Figure 2).

**Conclusion**

I commenced this article with an outline of many of the problems that are faced by students and teachers with middle school mathematics. I then outlined a range of strategies that I believe have the potential to make a difference to mathematics learning in these important years. For too long, mathematics learning has been seen by students as a negative experience, leading to the conclusion that they ‘can’t do mathematics.’ I am confident that with the right attitude, we can turn this around, leading to confident and capable
mathematics learners, as students continue with further study, and take their place in the workplace and in society.

Figure 2.

References


