SHRINE TO UNIVERSITY: MATHEMATICS IN THE CONSTRUCTED ENVIRONMENT

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A Mathematics Excursion

Excursions are taken for granted in most school subjects, but as teachers of mathematics we are often under such pressure to complete the curriculum that we are reluctant to undertake any activities which could be regarded as 'wasting time'. After all, writing as long ago as 1912 in her *Sourcebook of Problems for Geometry*, in which she explores the mathematics of architecture and design, Mabel Sykes (1912) admonished:

> At present there is a widespread tendency in education to substitute amusement for downright work. This temptation presents itself in varied and subtle forms to all teachers. Even the best teacher cannot escape. He must be continually on his guard lest his work call forth mere superficial attention and begin and end in entertainment. (Preface)

While Mabel's view may have reflected her times, most of us would now agree that mathematics needs to be enjoyable as well as rigorous and challenging. We need to develop in our students a more positive approach to mathematics and portray it as an important tool for exploring the world around us, as well as an exciting and evolving part of our cultural heritage. The wealth of interesting mathematics in our constructed environment provides an opportunity to extend mathematics beyond the four walls of the classroom and to emphasise some of the often-neglected visual aspects. Goldenberg, Cuoco and Mark (1998) express concern that the visual impoverishment of modern mathematics curricula in most countries has serious implications for the retention of students in mathematics courses:

> Some students who would like a visually rich mathematics never find out that there is one because they've already dropped out before they've had the chance to encounter any of the more visual elements. We lose not only potential geometers and topologists in this way, but all students who might enter mathematics through its visually richer domains and then discover other worlds, not as intrinsically visual, to which they can apply their visual abilities and inclinations (p.5).
>
> For some students, a visual approach may be absolutely essential. For such students, including many who consider themselves to be poor at mathematics, visual approaches are access. Thus, for many students, visualization and visual thinking serve not only as a potential hook, but also as the first opportunity to participate (p. 6).

Geometry in St. Kilda Road and Swanston Street

The route along St. Kilda Road and Swanston Street to the University of Melbourne provides a rich resource for both primary and secondary mathematics. At primary and junior secondary levels there is an opportunity to investigate angles, circles, polygons, tessellations, the Melbourne Central cone and geodesic dome and Petrus Spronk's Pythagoras-inspired *Architectural Fragment*, all of which link with the Space and Measurement strands of Levels 3, 4 and 5 of the Curriculum and Standards Framework (CSF II) (Board of Studies, 2000). Follow-up classroom activities can include construction of models and tessellations. The Gothic architecture of St. Paul's Cathedral, Melbourne Central cone, the Ansett A and the tessellating pentagons at the University of Melbourne provide interesting applications of geometry and trigonometry for the middle secondary years (CSF Level 6). At senior levels, the geometric sequence occurring in the levels of the Arts Centre spire, where the common ratio is a trigonometric function, represention of the beams at the base of the spire as three-dimensional vectors or modelling the Nautilus fountain as a logarithmic spiral may form the basis of challenging classroom-based problem-solving activities.

The Floral Clock

The floral clock (Figure 1) in the Queen Victoria Gardens on St. Kilda Road has an outer radius of 4 metres, with an inner circle of radius 2.5 metres (Figure 2). The lengths of the minute hand and the hour hand are 2.1 metres and 1.7 metres respectively. The clock is replanted several times each year, usually with a geometric design in the inner circle (see Figure 3). Depending on the design, approximately 4000 plants are needed for the inner design and 3000–3500 for the outer section. Students could design their own floral clock flower bed, an activity which may incorporate calculation of area and circumference of circles, circle designs, symmetry, estimation (or calculation) of areas, numbers of plants required and calculation of the cost of planting. It may even be possible to create the flower bed in the school grounds.

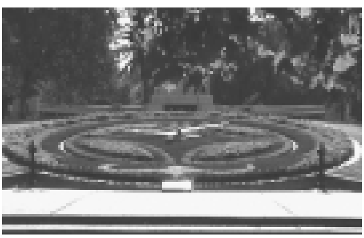


Figure 1. The Floral Clock.

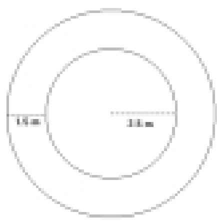


Figure 2. Clock dimensions.

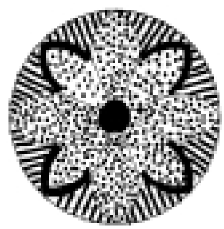


Figure 3. Flower design.

Arts Centre Spire

The base of the Arts Centre spire consists of twelve ‘hypar’ shells, or hyperbolic paraboloids (Figure 4), which are separated from each other by steel beams. The design engineers have identified the positions of the ends, or nodes, of these beams by means of three-dimensional coordinates, where the St. Kilda Road street level is taken to be zero on the *Z*-axis. Table 1 shows the coordinates of nodes A_1, B_1 and C_1 (see Figure 5).

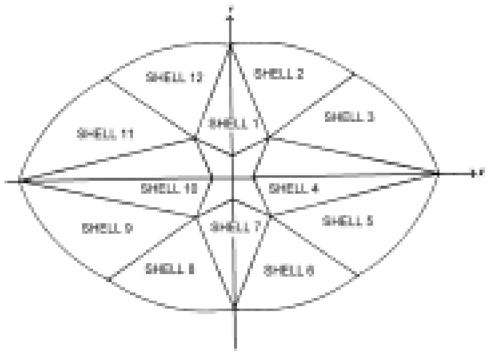


Figure 4. Plan view of the twelve shells of the Arts Centre spire base.

Table 1. Coordinates, measured in millimetres, of nodes A_1, B_1 and C_1

Node	***X***	***Y***	***Z***
A_1	9144	9144	40064
B_1	30666	23154	26105
C_1	50825	0	21554

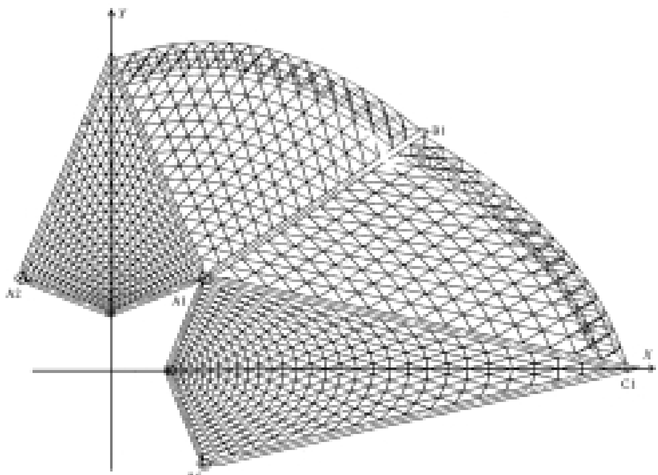


Figure 5. Plan view of shells 1,2,3 and 4 showing beams and nodes.

Using the coordinates, the lengths of the beams can be calculated using Pythagoras' theorem in three dimensions. For example, the beam A_1B_1 can be represented as the diagonal of a cuboid as shown in Figure 6. The length of A_1B_1 can then be calculated as follows:

$$
\begin{aligned}
l &= 30666 - 9144 \\
&= 21522 \text{ mm} \\
w &= 23154 - 9144 \\
&= 14010 \text{ mm} \\
h &= 40064 - 26105 \\
&= 13959 \text{ mm} \\
\text{Distance from } A_1 \text{ to } B_1 &= \sqrt{l^2 + w^2 + h^2} \\
&\approx 29229 \text{ mm} \\
&= 29.2 \text{ m}
\end{aligned}
$$

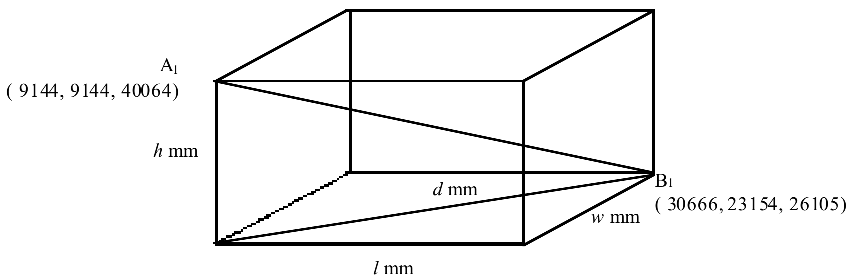


Figure 6. Three dimensional representation of points A_1 and B_1.

The coordinates can also be used to represent each of the nodes as a position vector. For example, the position vector of A1 is 9144 $\boldsymbol{i}$ + 9144 $\boldsymbol{j}$ + 40064 $\boldsymbol{k}$, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors in the X, Y and Z direction respectively. Each of the steel beams can then be represented as a 3-dimensional vector. For example, the beam A_1B_1 can be represented as the vector $\overrightarrow{A_1B_1}$.

$$
\begin{aligned}
\overrightarrow{A_1B_1} &= \overrightarrow{O_1B_1} - \overrightarrow{O_1A_1} \\
&= (30666\,\boldsymbol{i} + 23154\,\boldsymbol{j} + 26105\,\boldsymbol{k}) - (9144\,\boldsymbol{i} + 9144\,\boldsymbol{j} + 40064\,\boldsymbol{k}) \\
&= 21522\,\boldsymbol{i} + 14010\,\boldsymbol{j} - 13959\,\boldsymbol{k}
\end{aligned}
$$

The magnitude of vector $\overrightarrow{A_1B_1} = \sqrt{21522^2 + 14010^2 + 13959^2}$ mm

$= 29228.9$ mm. $= 29.2289$ m

The length of the steel beam connecting A_1 and B_1 is therefore approximately 29.2 m.

Similarly, vector $\overrightarrow{A_1C_1}$ can be shown to be 41681 $\boldsymbol{i}$ – 9144 $\boldsymbol{j}$ – 18510 $\boldsymbol{k}$, with magnitude 46514 mm. The angle θ between the two vectors can then be calculated:

If $\boldsymbol{a}$ and $\boldsymbol{b}$ are two vectors, where $\boldsymbol{a} = x_1\boldsymbol{i} + y_1\boldsymbol{j} + z_1\boldsymbol{k}$ and $\boldsymbol{b} = x_2\boldsymbol{i} + y_2\boldsymbol{j} + z_2\boldsymbol{k}$, the angle θ between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ may be calculated using $|\boldsymbol{a}|\,|\boldsymbol{b}|$ cos

$\theta = \boldsymbol{a.b}$ where $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$ are the magnitudes of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively and $\boldsymbol{a.b}$ is the scalar product.

$$\text{Hence } \cos\theta = \frac{\boldsymbol{a.b}}{|\boldsymbol{a}||\boldsymbol{b}|} = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{(x_1^2 + y_1^2 + z_1^2)(x_2^2 + y_2^2 + z_2^2)}}$$

$$= \frac{41681 \times 21522 - 9144 \times 14010 + 18510 \times 13959}{46514 \times 29229}$$

$$= 0.7556$$

$$\therefore \theta = 40°55'$$

Pedestrian Bridge at Southgate

The pedestrian bridge over the Yarra River at Southgate is a bow truss design in the shape of a parabola. Superimposed on the photograph of the bridge (Figure 7) are coordinate axes, with the origin at the vertex and the coordinates, in metres, of point A at one end of the arch. The equation for the parabolic arch can thus be determined, then graphed using a graphic calculator (Figure 8).

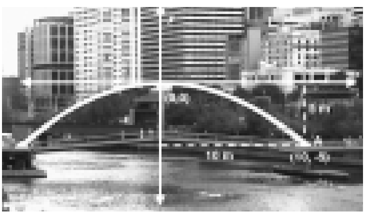


Figure 7. The pedestrian bridge at Southgate.

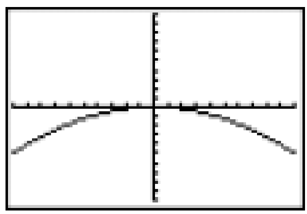


Figure 8. Graph of $y = -0.05x^2$.

Assuming a parabolic shape, the equation will be of the form:

$$y = ax^2$$

$$\text{Substitute } x = 10,\ y = -5$$

$$-5 = 100a$$

$$\therefore a = -0.05$$

$$\therefore y = -0.05x^2$$

St. Paul's Cathedral

St. Paul's Cathedral is built in the Gothic style which originated in 12th century Europe. Equilateral triangles form the basis of the simple Gothic arch and three-leafed trefoils while quatrefoils (Figure 9) are based on squares. Multifoils based on the pentagon, hexagon and heptagon are also to be found on the exterior of the Cathedral. Designs such as the equilateral arch and trefoil arch (Figure 10) and quatrefoils can be constructed using pencil and compass or dynamic geometry software, while at a more advanced level, algebraic relationships may be explored.

Figure 9. St. Paul's Cathedral: a quatrefoil.

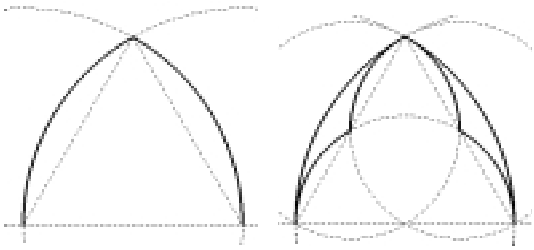
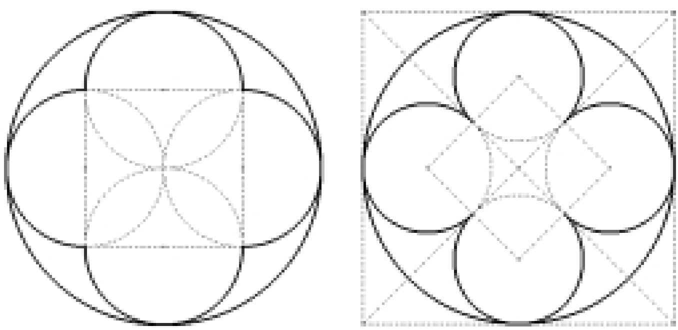


Figure 10. Constructing equilateral and trefoil arches.

Quatrefoils may be formed from intersecting circles (Figure 11a) or from tangent circles (Figure 11b). In the intersecting circles quatrefoil, it can be seen that the radius of the inner circles is half that of the outer circle. For the tangent circles quatrefoil, calculation of the radius of each inner circle is an appropriate task for Year 10 students involving Pythagoras' theorem, solving a quadratic equation and manipulation of surds.



(a) (b)

Figure 11. Quatrefoils: (a) Intersecting circles and (b) tangent circles.

If the radius of the large circle of the tangent circle quatrefoil is R and the radius of each small circle is r (see Figure 12), AO = BO = $R - r$ and AB = $2r$.

$$
\begin{aligned}
\therefore\ (R-r)^2 + (R-r)^2 &= (2r)^2 \\
\therefore\ 2\,(R^2 - 2Rr + r^2) &= 4r^2 \\
\therefore\ R^2 - 2Rr - r^2 &= 0 \\
\therefore\ R &= r\,(1+\sqrt{2}) \\
\text{i.e., } r &= \frac{R}{1+\sqrt{2}} = R\,(\sqrt{2}-1)
\end{aligned}
$$

Many more problems based on Gothic tracery are to be found in *A sourcebook of problems for geometry* (Sykes, 1912, recently republished by Dale Seymour Publications).

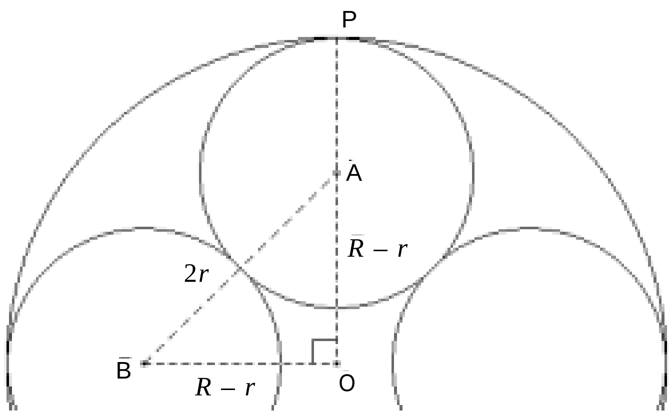
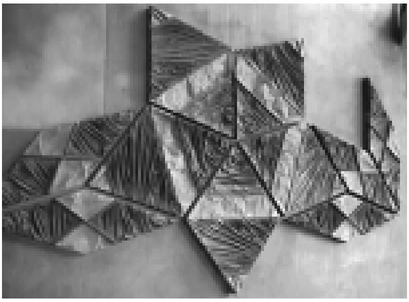


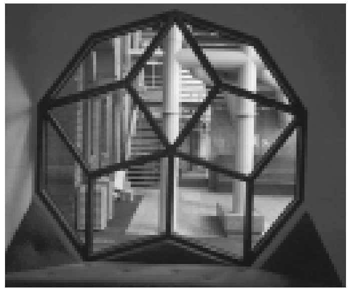
Figure 12. Relationship between radii of outer and inner circles in the tangent circles quatrefoil.

Storey Hall, RMIT

In their redesigning of Storey Hall in 1996, Melbourne architects, Ashton Raggatt McDougall, combined history with geometry to create a 'mathematical wonderland' (Figure 13). The dominant purple and emerald green are symbolic of the building's former use by the Suffragette movement early in the nineteenth century, and later by the Irish Hibernian Society. Spreading chaotically over the interior and exterior of the building are tessellating rhombuses based on Sir Roger Penrose's aperiodic tiling of two simple rhombuses. Penrose discovered that the two rhombuses (Figure 14) will tessellate infinitely without repeating exactly the same pattern so the tessellation may be likened to an irrational number.



(a) Exterior wall



(b) Window at rear of Storey Hall Gallery

Figure 13. Tessellating Penrose rhombuses at Storey Hall, RMIT.

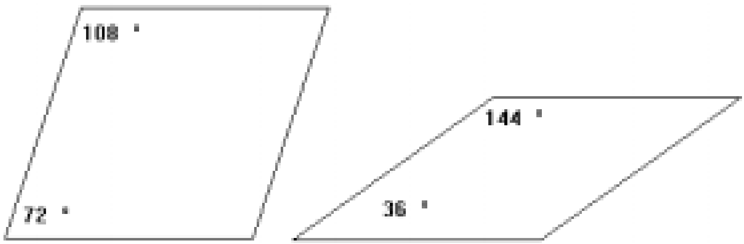


Figure 14. Penrose “fat” and “thin” rhombuses.

The rhombuses tessellate to form decagons (Figure 15) and these in turn overlap on thin rhombuses to form an aperiodic tiling. Superimposed over the rhombuses is a green pentagon pattern related to the allowed arrangements of fat and thin rhombuses to ensure an infinite tiling. It is possible to arrange the tiles in such a way that a point is reached when neither tile can be added.

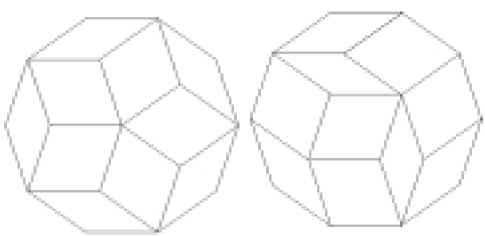


Figure 15. Decagons formed from tessellating Penrose rhombuses.

On one level, students may create tessellations, using different colours for the fat and thin rhombuses, while at another level they may explore the ratio of fat to thin rhombuses in the tessellation (see Figure 16). As the number of tiles in the tessellation increases, the ratio of fat to thin rhombuses approaches the golden ratio, which is, of course, an irrational number.

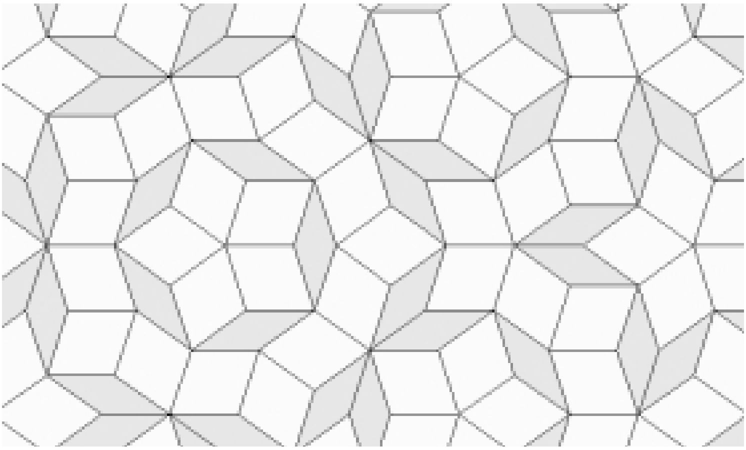


Figure 16. Tessellating Penrose rhombuses.

Ansett A

Outside the Ansett building at 501 Swanston Street, opposite the Melbourne City Baths, is a sculpture in the shape of a tetrahedron, or triangular pyramid (Figure 17), representing the former Ansett logo which was adopted by Ansett Airlines of Australia in 1969. The current logo of Ansett Australia is a stylised gold letter “A” with a seven-pointed star. The original logo, as depicted in the sculpture, is an isosceles triangle but the division of the triangle to form the letter A provides an interesting study of similar triangles (Figure 18). Given that $\angle BAM = 27°$ and $\angle BDO = 90°$, all the other angles can be calculated. Triangles ABM, AOD and BOM are similar and triangles OBD and ACD are similar. O is the orthocentre, that is, the intersection of the altitudes, of triangle ABC. An investigation of this triangle could be extended to an exploration of other points of concurrency in triangles.

Figure 17. Ansett A sculpture.

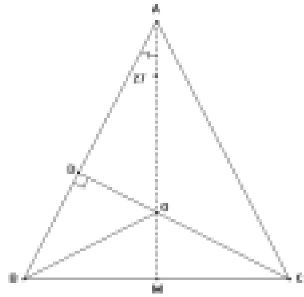


Figure 18. Geometric structure of the Ansett A.

Tessellating pentagons

Between the Old Geology building and the Department of Mathematics and Statistics at the University of Melbourne there is a courtyard of tessellating pentagon-shaped paving stones (Figure 19). While regular pentagons will not tessellate, this pentagon is one of several irregular pentagons which tessellate. The landscape architects, Paterson and Pettus, based their design on Marjorie Rice's tiling which they found in *The Emperor's New Mind* (Penrose, 1989, p. 133). Eight of the pentagons tessellate by rotation and reflection (Figure 20) to form blocks which tessellate by translation (Figure 21). The tessellation therefore provides an interesting illustration of transformations. Further pentagon tessellations are discussed in *The Penguin Dictionary of Curious and Interesting Geometry* (Wells, 1991, pp. 177–179).

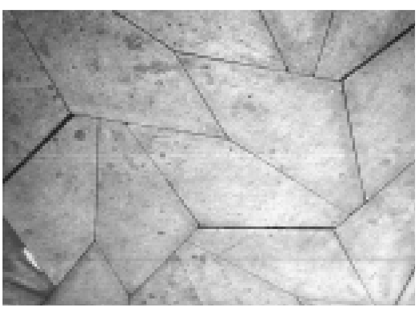


Figure 19. Tessellating pentagon paving stones.

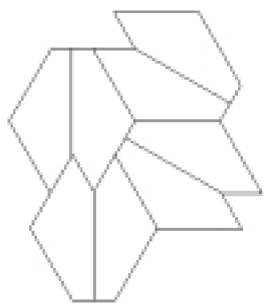


Figure 20. Pentagons are reflected and rotated.

Figure 22 shows the angle sizes and the ratio of the five sides. Combinations of angles which add to either 180° or 360° allow the pentagon to tessellate.

After constructing the pentagon using ruler, pencil and protractor or dynamic geometry software, the pentagon can then used as a template for creating a tessellation. Determining the angle sizes from a study of the tessellation and calculating the fifth side, given the ratio of the four sides 1:4:4:2, are appropriate tasks for Year 9–10 students.

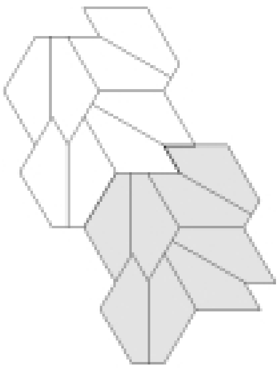


Figure 21. Tessellating blocks of 8 pentagons.

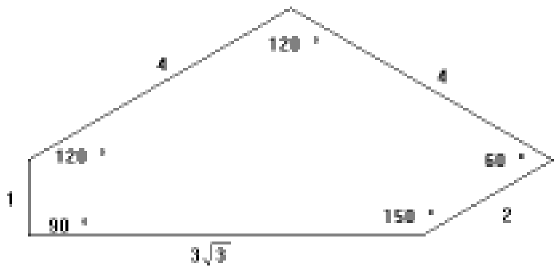


Figure 22. Angles and ratios of sides in the pentagon paving stones.

Conclusion

These examples of the many applications of geometry along the St. Kilda Road – Swanston Street route demonstrate the potential of using the constructed environment as a mathematics resource. Integrated into the mathematics curriculum, these activities may help to develop spatial reasoning skills as well as creating a more positive attitude to mathematics.

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